Maine's Advanced Placement Incentive Program: Curriculum Analysis Report

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I: Executive Summary

Introduction

A Maine Department of Education vision for all students is the opportunity to experience rigorous coursework throughout their educational years, with the support to meet high standards and be prepared for post secondary education. The school mathematics curriculum is the key determinant of what students have an opportunity to learn and what they do learn. An effective mathematics curriculum must be coherent with interconnected topical strands of both content and process.

Since a well articulated, cohesive mathematics program lays an important foundation for student success in mathematics, the Maine Department of Education was interested in learning more about common curriculum used in Maine. This report is the result of a ten month review and analysis by the Maine Mathematics and Science Alliance (MMSA) to determine the potential efficacy of the selected curriculum programs for helping students achieve high standards and gain access to advanced study in mathematics. The measure for advanced mathematics in this report is the Advanced Placement Mathematics Program® (College Board, 1955).

The analysis process included an examination of instructional practice and mathematics content. An in-depth analysis was completed for three commonly used mathematics curriculum programs at the middle and high school levels. The curriculum materials for this analysis include two contemporary programs and one traditional program for high school and for middle school. These selections were made from among several recommendations by Maine educators and the Maine Department of Education as common, although not the only, mathematics curriculum used in Maine.

The analysis is designed to:

i) Align the content of the material to National Standards for School Mathematics (NCTM Standards), the Maine Learning Results, and the College Board Standards for College Success

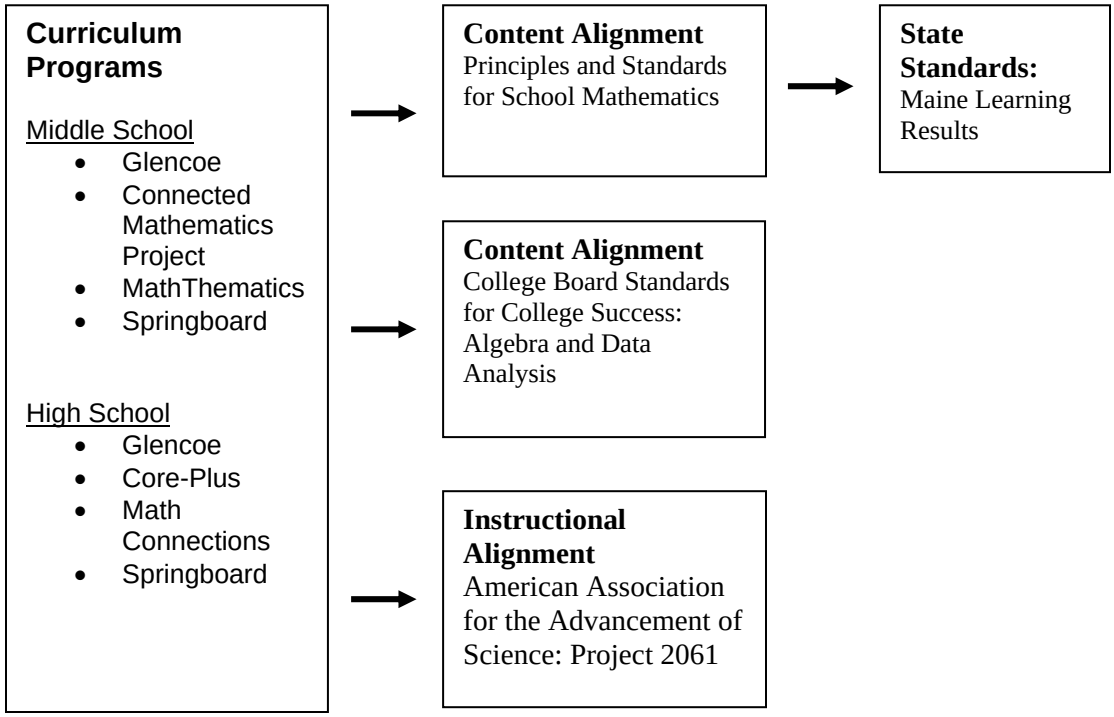
ii) Compare the content and instructional alignment to the SpringBoard material and

iii) Provide a structure for use by vertical teams in analyzing their existing program(s) to make informed decisions about use of resources and placement of students.

Findings:

The findings from this analysis include a content alignment with standards and an instructional alignment with researched-based principles for learning and teaching mathematics (see Figure 1: Analysis Overview). An additional review was conducted of the Springboard materials published by the College Board. Each area is discussed here with a final summary.

Figure 1: Analysis Overview



Content Alignment Maine Learning Results and National Standards:

The content alignment to standards was conducted to determine if the curricula, as written, provide the opportunity for Maine students to achieve the identified standards. The five content standards of the national standards are represented in each of the Middle Level mathematics programs analyzed for this report: contemporary programs, Connected Mathematics Project and Math Thematics and a more traditional program, Glencoe's Mathematics: Applications and Concepts. With the exception of Glencoe's geometry coverage and Math Thematics' measurement coverage, all other content area coverage was above the balance recommended by standards from the National Council of Teachers of Mathematics. The content area coverage for each of the three programs was at or above the balance of Maine's mathematics standards as determined by the Department of Education's Balance of Representation (June, 2003).

The five content standards of the national standards are also represented in each of the high school programs analyzed for this report: contemporary programs, Math Connections and Contemporary Math in Context and a more traditional program Glencoe's Algebra I, Geometry, Algebra II. With the exception of Glencoe's numbers & operations coverage, all other content area coverage was at or above the balance

recommended by standards from the National Council of Teachers of Mathematics. The content area coverage for each of the three programs was at or above the balance of Maine's mathematics standards as determined by the Department of Education's Balance of Representation with the exception of the coverage of data analysis & probability in the Glencoe text.

In many areas, the contemporary programs show greater content area coverage than the recommendations of the National Council of Teachers of Mathematics, Maine's Balance of Representation, and the more traditional series. This appears to result from the contemporary programs' integration of topical areas by using a problem centered approach to mathematics rather than addressing the topics in isolation.

Content Alignment: *College Board Standards for College Success*

The alignment analysis to *College Board Standards for College Success* focused on Algebra and Data Analysis & Probability because the concepts within these two standards are the foundational ideas necessary for success in the AP Calculus and AP Statistic courses.

Within each of the strands, three to six levels represent a progressive framework of skills and knowledge. Two to five performance indicators further clarify what students should know and be able to do at that particular level. The primary instructional goals of each lesson of the three middle level and the three high school programs were analyzed and aligned at the performance expectation level of the College Board Standards for College Success. In many areas, the contemporary programs show greater Performance Expectation coverage than the more traditional program.

The alignment analysis of the College Board Standards for College Success allows vertical teams using a combination from among the six programs to make data driven decisions on supplemental programs and units such as the Springboard program as well as in planning for key concepts and ideas to be taught in a pre-calculus setting. Vertical teams not using any combination of these six programs can follow the analysis procedure to align the programs currently in use.

Instructional Alignment:

The instructional alignment determines the extent to which the materials' reflects what is known about student learning and effective teaching. For this process, six categories from the Project 2061 Curriculum Analysis procedure were used for alignment purposes, with each of the 6 categories broken into 2-6 criteria. An overall rating of each of the categories provides a typical alignment for each of the programs.

Analysis of the contemporary programs reveal strengths that include the categories related to promoting student thinking about mathematics and assessing student progress. The ratings in the areas of specifying prerequisite knowledge and providing practice were similar for all programs. The ratings in other categories and areas varied from program to program with none of them having a strong alignment to the criteria of providing teacher content support or identifying student misconceptions.

SpringBoard Comparison:

SpringBoard, published by the College Board, provides an opportunity for supplementing a district program lacking in either content or process. The analysis of the SpringBoard material with the instructional criteria resulted in a strong alignment in the areas of providing a variety of contexts, encouraging students to explain their reasoning, and providing informal and formal assessment opportunities. These strengths are similar to those of the contemporary programs analyzed for the report. In the other areas, Springboard's content and instructional alignment varies in comparison to the six programs.

The College Board, the sponsor of the Advanced Placement program, has been supporting content coordination among the grades through vertical teaming. One of the primary goals of a vertical team is to develop and implement a coherent curriculum so all students have access to a high quality mathematics program. This program would then result in improved academic performance for all students in earlier grades than when traditional mathematics might be available in a sequence introducing skills and concepts needed for success in AP and other challenging courses. By identifying and developing Pre-AP strategies and vertically aligning curriculum toward the discipline-based AP standards, the opportunity for learning higher level mathematics is increased. (College Board, 2004)

Since none of the commercial programs analyzed provide complete coverage, this in-depth analysis of both content and process was designed to provide additional support in helping Maine teachers and districts make informed decisions about mathematics curriculum programs which will provide all students with the best opportunity for advanced learning in mathematics.

Summary

The contemporary mathematics programs tended to be better aligned than the traditional series reviewed in this examination in both the content and instructional review. This result is not an endorsement of a particular program, but rather a statement of the advantages of curriculum in helping students achieve higher levels of mathematics.

It is apparent that a curriculum mathematics program alone will not meet all the criteria for an effective mathematics program. The teachers' knowledge of mathematics, curriculum foundation, instructional methods, and student motivation are important factors in the overall success of any mathematics program. This study is focused on two of the above factors that, if used in an effective manner, could help teachers make improvements in their students' mathematics achievement.

II. Introduction

Excellence for all students in Maine is a goal of all Maine educators. One way to accomplish excellence is to implement research supported materials and a coherent curriculum. This is the case for the Advance Placement (AP) programs as they contribute greatly to advanced students in high schools. High school curricula are often affected by what is in the AP program in a school. The challenge at the middle and high school level is to prepare all students for more rigorous mathematics and science programs while at the same time providing advanced offerings through an advanced study program. In addition to this challenge, Maine schools are beginning to examine practice and specific curriculum programs for alignment to state standards as they are held to a higher level of accountability through Maine's Comprehensive Assessment System.

This report is the result of a ten month review and analysis process developed at the Maine Mathematics and Science Alliance (MMSA) in collaboration with the Maine Department of Education (DOE) and Maine teachers. The analysis and subsequent report is a part of a larger initiative of the Maine Department of Education through Maine's Advanced Placement Incentive Program. The goals of that program include the following:

- Increase the number of students, especially low-income students, taking AP courses.
- Increase the rigor of coursework in at least grades 6-AP that is connected with Maine's Learning Results
- Students will have access to rigorous curriculum, encouraging secondary schools to better prepare more students for Advanced Placement courses.

Since a well articulated, cohesive mathematics program lays an important foundation from which to examine instructional practice, an in-depth analysis was done of 3 commonly used mathematics curriculum programs at both the middle and high school level. The analysis is designed to i) align the content of the material to National Mathematics Standards, the Maine Learning Results, and the College Board Standards ii) compare the content and instructional alignment to the SpringBoard material and iii) provide a structure for use by vertical teams in analyzing their existing program(s) to make informed decisions about use of resources and placement of students.

A school mathematics curriculum is a key determinant of what students have an opportunity to learn and what they do learn. A mathematics curriculum must be coherent with interconnected topical strands of both content and process. As stated in the AP Calculus teacher's Guide, "Although most teachers in today's AP classrooms learned calculus through lectures with little class participation, it is now widely conceded that there are better ways to foster the learning of mathematics. College professors committed to calculus reform are encouraging more cooperative learning, group projects, student discovery, computer lab work, and classroom dialogue. In a direct connection to this method of learning and teaching, the goals of AP Calculus are constructed to be multirepresentational (geometric, analytic, numeric, verbal), emphasizing broad concepts and widely applicable methods, using technology, and presenting calculus as a cohesive

whole." (College Board, 2004) The AP Statistic's guide follows suit by stating, "the course lends itself naturally to a mode of teaching that engages students in constructing their own knowledge....this approach gives students ample opportunity to think through problems, make decisions, and share questions and conclusions with other students as well as with the teacher" (College Board,2004). This same instructional philosophy and approach needs to be incorporated in classes prior to an AP level course.

While helping more students get into advance classes may be possible, the more important question that guides this analysis is, "How can high school science and mathematics education at both the introductory and advanced levels be improved so that a larger number of students will have access to advanced study and a realistic chance of succeeding once enrolled?" (NRC,2002 pg. 23). One way to support this more inclusive vision is to know something about the curriculum materials that teachers regularly use. This analysis at both the high school and middle school level begins to examine the opportunity to learn mathematics in each of several curricula.

Much of the research conducted in the last twenty years has added to our understanding of human cognition. This research has helped educators develop some underlying principles for learning for all students. The National Research Council's committee on advanced study organized their work around seven principles and believes that advanced study in K-12 schools should also use these principles. They are:

1. Learning with understanding is facilitated when new and existing knowledge is structured around the major concepts and principles of the discipline.
2. Learners use what they already know to construct new understanding.
3. Learning is facilitated through the use of metacognition strategies that identify, monitor and regulate cognitive process.
4. Learners have different strategies, approaches, patterns of abilities, and learning styles that are a function of the interaction between their heredity and their prior learning experiences.
5. Learners motivation to learn and sense of self affects what is learned, how much is learned, and how much effort will be put into the learning process.
6. The practices and activities in which people engage while learning shape the what is learned.
7. Learning is enhanced through socially supported interactions. (NRC 2002)

Students' educational opportunities and achievement are tied to the beliefs and values of those who teach them. Offering more advanced content to all students is one way to send a message that they are valued and that they are expected to and can attain access to higher education. The Maine DOE vision for all students is that they will have the opportunity to experience rigorous coursework throughout their educational years, have the support to meet high standards and be prepared for post secondary education.

One factor limiting access to postsecondary education is being placed outside the top academic track in high school. The implementation of a high quality, coherent, and connected curriculum program can provide all students the opportunity to learn mathematics with understanding, laying an important foundation for success in future courses. Teachers in Maine use a variety of mathematics curriculum programs, but in the

last five years many schools have adopted materials more recently developed by researchers and math educators. These “contemporary” materials are sometime called “standards based”. The selection of curriculum materials for this analysis includes two contemporary programs and one traditional program for both high school and middle school, totaling six different curriculum programs. These selections were made from among several recommendations by educators and the Maine DOE as common, although not the only, mathematic curricula used in Maine.

This report is divided into several sections including a description of the analysis process, overview of the curriculum materials analyzed, comparisons of content, and comparison of instructional criteria. A comparison is made between the content in the mathematics materials and the requirements of the National Standards, the Maine Learning Results and the Advanced Placement programs. A comparison is made between instructional, curricular, and assessment strategies in the materials that can best support student learning as researched by the American Association for the Advancement of Science (AAAS).

III: Description of Analysis Procedure

"The American Association for the Advancement of Science (AAAS) founded Project 2061 in 1985 to help all Americans become literate in science, mathematics, and technology. In *Science for All Americans*, Project 2061 set out recommendations for what all students should know and be able to do in science, mathematics, and technology by the time they graduate from high school. *Benchmarks for Science Literacy*, published in 1993, translated the literacy goals in *Science for All Americans* into learning goals or benchmarks for grades K–12. Knowing that for many mathematics teachers, the textbook is their primary guide to implementing the curriculum, Project 2061 began an evaluation of mathematics textbooks for the alignment to the learning goals outlined in *Benchmarks for Science Literacy*. The analysis was based on three propositions: First, good textbooks can play a central role in improving mathematics education for all students; second, the quality of mathematics textbooks should be judged mainly on their effectiveness in helping students achieve important mathematics learning goals for which there is a broad national consensus; and third, an in-depth analysis of much more than a textbook's content coverage would be required to evaluate whether there is potential for students' actually learning the desired subject matter."(AAAS, 2000). A detailed description of the evaluation process and results can be found on the Project 2061 website. Although MMSA used the process and criteria as outlined by AAAS, we do not claim that the extent of the analysis is at the same level and depth.

MMSA adapted the Project 2061 procedure in the following way:

- Step 1: Identify specific learning goals to serve as the basis for the analysis.

  The National Council of Teachers of Mathematics' content standards rather than the *Benchmarks* were used as the source for identifying learning goals. Each of the NCTM content standards is broken into goal areas that stay consistent throughout the K-12 document. The NCTM expectations for each of the grade span goal areas were reduced/merged to a manageable level for this analysis, (see Appendix A for list of content standards, goals, and expectations).

- Step 2: Analyze the curriculum materials for alignment between content and the selected learning goals.

  In the analysis, each lesson/activity was aligned to a specific NCTM goal area and expectation of the content standard. The goal area is used for the analysis with the expectations serving as criteria for alignment. The purpose here is to determine whether the content in the material matches specific learning goals, not just whether the topic headings are similar.

- Step 3: Analyze the curriculum materials for alignment between instruction and the selected learning goals.

  This step involves estimating the degree to which the written materials reflect what is known generally about student learning and effective teaching and, more important,

the degree to which they support student learning of the specific knowledge and skills for which a content match has been found.

Six of the areas from Project 2061's procedure were used in this analysis:

> **Category I: Identifying a Sense of Purpose.** Part of planning a coherent curriculum involves deciding on its purposes and on what learning experiences will likely contribute to achieving those purposes. Three criteria are used to determine whether the lesson/activity conveys a unit purpose and a lesson purpose and justifies the sequence of activities.
>
> **Category II: Building on Student Ideas about Mathematics.** Fostering better understanding in students requires taking time to attend to the ideas they already have, both ideas that are incorrect and ideas that can serve as a foundation for subsequent learning. Four criteria are used to determine whether the lesson/activity specifies prerequisite knowledge, alerts teachers to student ideas, assists teachers in identifying student ideas, and addresses misconceptions.
>
> **Category III: Engaging Students in Mathematics.** For students to appreciate the power of mathematics, they need to have a sense of the range and complexity of ideas and applications that mathematics can explain or model. Two criteria are used to determine whether the lesson/activity provides a variety of contexts and an appropriate number of firsthand experiences.
>
> **Category IV: Developing Mathematical Ideas.** Mathematics literacy requires that students see the link between concepts and skills, see mathematics itself as logical and useful, and become skillful at using mathematics. Six criteria are used to determine whether the lesson/activity justifies the importance of learning goal ideas, introduces terms and procedures only as needed, represents ideas accurately, connects learning goal ideas, demonstrates/models procedures, and provides practice**.**
>
> **Category V: Promoting Student Thinking about Mathematics.** No matter how clearly lesson/activity s may present ideas, students (like all people) will devise their own meaning, which may or may not correspond to targeted learning goals. Students need to make their ideas and reasoning explicit and to hold them up to scrutiny and recast them as needed. Three criteria are used to determine whether the lesson/activity encourages students to explain their reasoning, guides students in their interpretation and reasoning, and encourages them to think about what they've learned.
>
> **Category VI: Assessing Student Progress in Mathematics.** Assessments must address the range of skills, applications, and contexts that reflect what students are expected to learn. This is possible only if assessment takes place throughout instruction, not only at the end of a chapter or unit. Three criteria are used to determine whether the lesson/activity aligns assessments with the learning goals, assesses students through the application of learning goal ideas, and uses embedded assessments. (AAAS, 2000)

As noted, each of the categories is broken into 2-6 criteria, which are further detailed using indicators. The categories were used for the instructional analysis with the number of indicators aligned serving as criteria for a low, moderate or high alignment rating (see Appendix B for list of Categories, Criteria, and Indicators).

- Step 4: Summarize the relationship between the curriculum materials being evaluated and the selected learning goals and instructional categories.

A statement of strengths and weaknesses of each of the programs in both content and instructional alignment is detailed in Section V and VI of this report.

IV. Overview of Programs Analyzed

Middle School Programs

SpringBoard
Publisher: College Board
Edition: 2004

This series is designed for grades 6 & 7. The books are labeled Middle School Math I and Middle School Math II. Each book consists of instructional materials, called model instructional units. Each model instructional unit consists of a series of guided questions broken into Parts. The number of parts range from 1 to 6 with the majority of the units consisting of 2-3 parts. The instructional time per part ranges from 40-60 minutes. SpringBoard units teach real world problem-solving based on the critical thinking skills described in the College Board Standards for College Success. Problem solving and collaborative learning strategies are embedded within the units.

The SpringBoard instructional resources are intended to supplement the texts and resources currently in use. Each of the books contains enough material for 2 units per month. Both the order and the number of units used is based on instructional needs of the students.

In a typical unit, students work through several parts of the instructional unit that include a theme and problems which are solved over several days.

Connected Math Project (CMP)
Publisher: Prentice Hall
Edition: 2002

This series is designed for grades 6,7, and 8. Each grade level consists of 8 units each named for the content/context within the unit. Each unit contains 4 to 7 investigations and each investigation contains 1 to 5 problems that the teacher and students explore in class

In a typical unit, students (1) respond to a set of focusing questions to pique curiosity about what they will learn, (2) read the Mathematical Highlights section, which lists goals, (3) work through several investigations that include a theme and problems, which are solved over several days and often in groups, (4) do homework from an Applications-Connections-Extensions section, (5) respond to questions in a Mathematical Reflections Summary, and (6) for some units, complete a final project that is launched at the beginning**.**

MathThematics
Publisher: McDougal Littell
Edition: 2002

This series is designed for grades 6, 7, and 8. The books are labeled Book 1, Book 2, and Book 3. Each book consists of eight modules. Each module has four to six sections. There are one to three explorations per section.

In a typical module, students (1) engage in a motivating activity that sets the stage for the section, (2) work through a series of activities learning mathematical concepts and skills and solving problems, (3) review what they've learned on a summary page, and (4) practice and apply skills with a series of exercises.

Mathematics: Applications and Concepts
Publisher: Glencoe
Edition: 2004

This series is designed for grades 6, 7, and 8. The books are labeled Course I, Course 2, and Course 3. Each book consists of 5-6 Units with 1-3 chapters per unit. Each chapter has 7-9 sections.

In a typical chapter, students study the context for the mathematics in the chapter and participate in a chapter project for applying the mathematics**.** Each chapter also includes activities engaging students in estimation strategies, decision making, cooperative learning, and mental math. In each lesson, students (1) read the objectives of the lessons, which may be followed by a short discussion of a real-life context for the mathematics, (2) work through a series of examples, and (3) apply their understanding of the objectives through guided practice and independent practice, including mixed review, problem solving, and applications.

High School Mathematics

SpringBoard
Publisher: Springboard
Edition: 2004

This high school series is designed for grades 9-12. The books are labeled High School Math I and High School Math II. Each book consists of instructional materials, called model instructional units. Each model instructional unit consists of a series of guided questions broken into Parts. The number of parts range from 1 to 6 with the majority of the units consisting of 2-3 parts. The instructional time per part ranges from 40-60 minutes. SpringBoard units teach real world problem-solving skills based on the critical thinking skills described in the College Board Standards for College Success. Problem solving and collaborative learning strategies are embedded within the units.

The SpringBoard instructional resources are intended to be supplemental material to the texts and resources currently in use. Each of the books contains enough material for 2 units per month. Both order and number of units used is based on instructional needs of the students.

In a typical unit, students work through several parts of the instructional unit that include a theme and problems which are solved over several days.

Core-Plus(Contempory Mathematics in Context)
Publsiher: Glencoe
Edition: 2003

This integrated series is designed for grades 9-12. It is a four-year curriculum that includes Courses 1-4. (Course 4, which is designed to prepare students for various mathematics and science undergraduate programs, was not analyzed for this project.) Each course consists of two parts (A and B) and a Capstone, a two-week project-oriented activity relevant to the complete course. Each course has seven units, with each unit consisting of three to five investigations.

In a typical unit, students engage in (1) Launch (Think About This Situation) - a whole-class discussion establishing a context for the lesson; (2) Explore - a small group investigation of more focused real-world problems; (3) Share and Summarize (Checkpoint) - a whole-class discussion enabling groups to summarize results of investigations and construct a shared understanding of important concepts, methods, and approaches; and (4) Apply - a task to be completed individually to reinforce understandings gained in the lesson.

Math Connections
Publisher: It’s About Time
Edition: 2000

This integrated series is designed for grades 9-11. Each grade level is divided into two books, a and b. The books are labeled 1a, 1b, 2a, 2b, 3a, and 3b. Each book is divided into chapters, which are divided into several sub-sections.

In a typical chapter, students read a profile about an individual who uses mathematics in his or her everyday work. In each section of the chapter, students (1) read expected learning outcomes; (2) are introduced to a concept by thinking about what they already know, which prompts discussion; (3) read commentary and explanations to support the discussion; and (4) answer questions in the section's problem set.

Algebra 1, Geometry, Algebra 2
Publisher: Glencoe
Edition: 2003

This series is designed for grades 9-11

In a typical chapter, students read the objectives, explore topics and situations that support the importance of mathematics, and engage in a chapter project. In each lesson, students (1) read what they will learn and why it is important; (2) explore commentary on demonstrations of the concepts or skills; (3) complete sample problems; and (4) express their understanding of concepts/skills by reading, writing, modeling, discussing, and doing practice problems. Each chapter ends with a highlights section and a study guide and assessment section, which include skills and concepts, applications and problem solving, alternative assessments, and, in some chapters, long-term investigation problems.

V: Comparison of Program Content

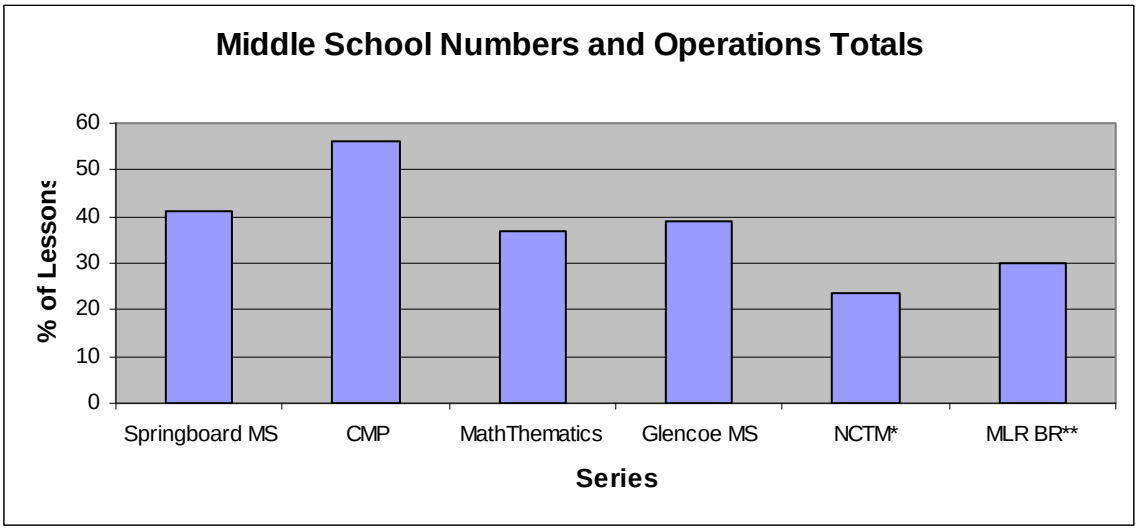
The analysis of the middle and high school mathematics programs was of the written material. The analysis does not include information obtained through professional development or additional support material available on the web or through the publisher. The delivery of the lesson is an important consideration as worthwhile lessons alone are not sufficient for effective teaching. Teachers must also decide what aspects of the lesson to highlight, how to organize and orchestrate the work of the students, what questions to ask, and how to support student thinking (NCTM, 2000). The SpringBoard program, although supplemental material, is included in the analysis to help teams make informed decisions on unit choice and order. The intent of the comparison of content across programs is not to rank or highlight a particular program or programs but to enable educators to use the data to provide all students access to rigorous curriculum.

Also included in the comparison of content is NCTM's growth expectations and Maine's Balance of Representation. The estimated NCTM percentages (Appendix C) highlight the growth of expectations across the grades. It is not expected that every topic will be addressed each year. Rather, students will reach a certain depth of understanding of the concepts and acquire certain levels of fluency with the procedures by prescribed points in the curriculum, so further instruction can assume and build on this understanding and fluency (NCTM,2002). The Balance of Representation percentages (Appendix D) map the "topography" of each discipline, identifying the relative mountains and valleys within the landscape of the *Learning Results*. The procedure is guided by knowledge of the discipline, knowledge of curriculum and instruction, and knowledge of students and developmental considerations.(DOE, 2002)

Middle Level

Numbers and Operations (NCTM Standard)

The NCTM content standard, Numbers and Operations, encompasses two of the Maine Learning Results Standards, Number Sense and Computation. The chart below shows the percent of lessons aligned to NCTM's Numbers and Operations Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



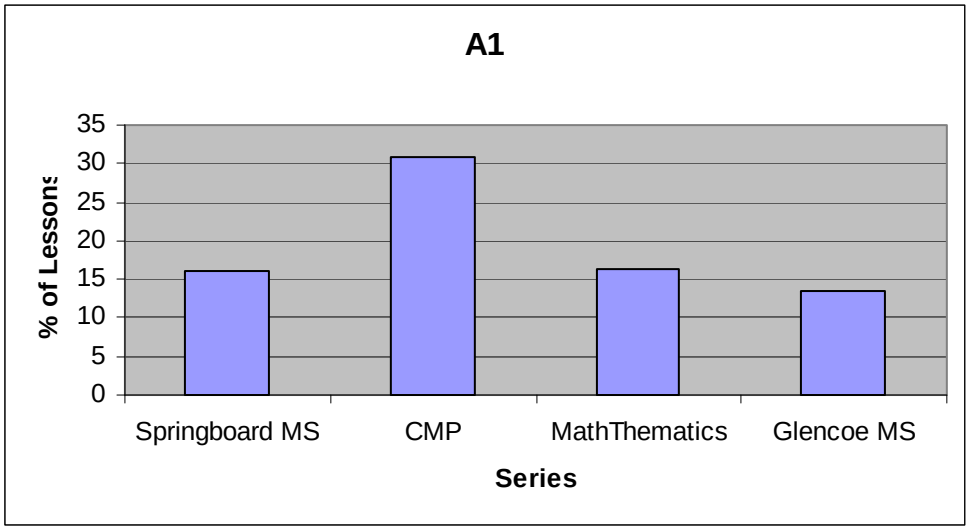
* NCTM % are approximate (Appendix C)
** Maine Learning Results Balance of Representation (Appendix D)

A breakdown of the Numbers and Operations alignment follows with the lesson alignment for the following three goal areas:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates

Each of the Number and Operations Goal Areas corresponds to the Maine Learning Results and the College Board Standards for College Success.

Numbers and Operations Goal Area 1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems (NCTM Goal Area Labeled A1 for this Analysis)



A1 Alignment to Maine learning Results and College Board Standards:

Maine Learning Results:

A. Numbers and Numbers Sense Grades 5-8

A1 Use numbers in a variety of equivalent and interchangeable forms

A3 Apply concepts of ratios, proportions, percents, and number theory (e.g., primes, factors, and multiples) in practical and other mathematical situations.

College Board Standards for College Success

Standard: Numbers and Operations

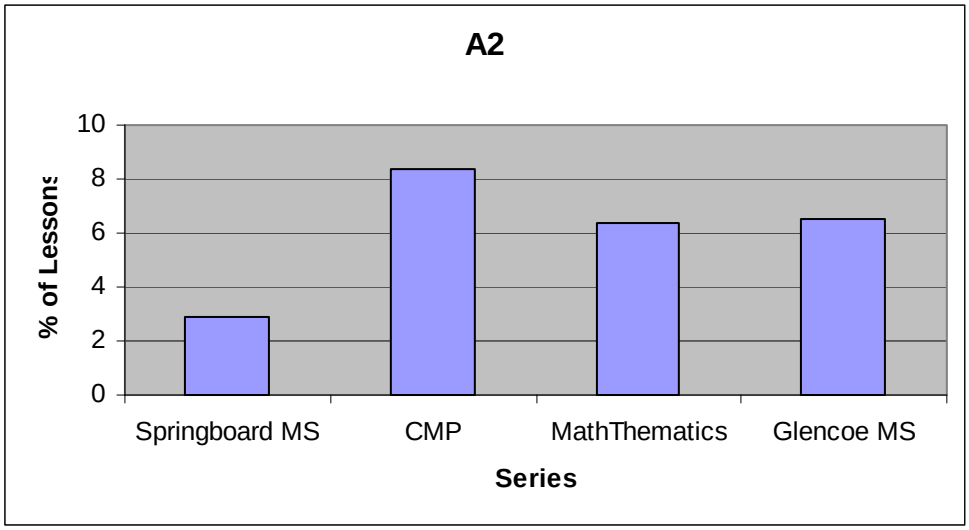
Strand: Number

Level 1 Performance Expectation 1-4

Level 2 Performance Expectation 1-3

Level 3 Performance Expectation 1-4

Numbers and Operations Goal Area 2: Understand meanings of operations and how they relate to one another (NCTM Goal Area Labeled A2 for this Analysis)



A2 Alignment to Maine learning Results and College Board Standards:

Maine Learning Results:

A. Numbers and Number Sense Grades 5-8

A2 Demonstrate understanding of the relationships among the basic arithmetic operations on different types of numbers

College Board Standards for College Success

Standard: Numbers and Operations

Strand: Operations

Level 1 Performance Expectation 1-4

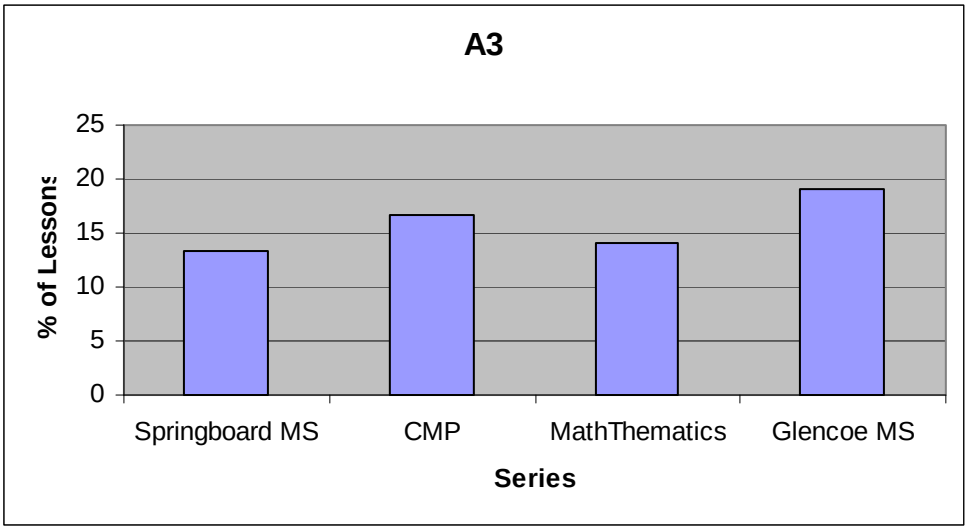
Level 2 Performance Expectation 1-4

Level 3 Performance Expectation 1-2

Strand: Computation and Estimation

Level 1 Performance Expectation 2

Numbers and Operations Goal Area 3. Compute fluently and make reasonable estimates (NCTM Goal Area Labeled A3 for this Analysis)



A3 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

B. Computation

B1. Compute and model all four operations with whole numbers, fractions, decimals, sets of numbers, and percents, applying the proper order of operations.

B2. Create, solve, and justify the solution for multi-step, real-life problems including those with ratio and proportion.

College Board Standards for College Success

Standard: Numbers and Operations

Strand: Computation and Estimation

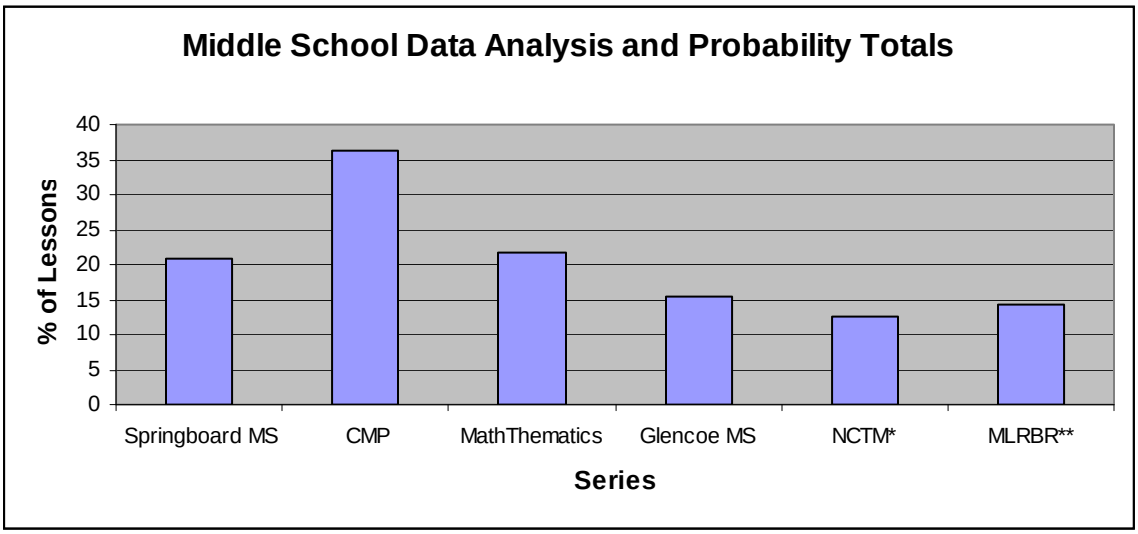
Level 1 Performance Expectation 1,3-5

Level 2 Performance Expectation 1-4

Level 3 Performance Expectation 1-4

Data Analysis and Probability

The NCTM content standard, Data Analysis and Probability, encompasses two of the Maine Learning Results Standards, Data Analysis& Statistics and Probability. The above chart shows the percent of lessons aligned to the Data Analysis and Probability Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



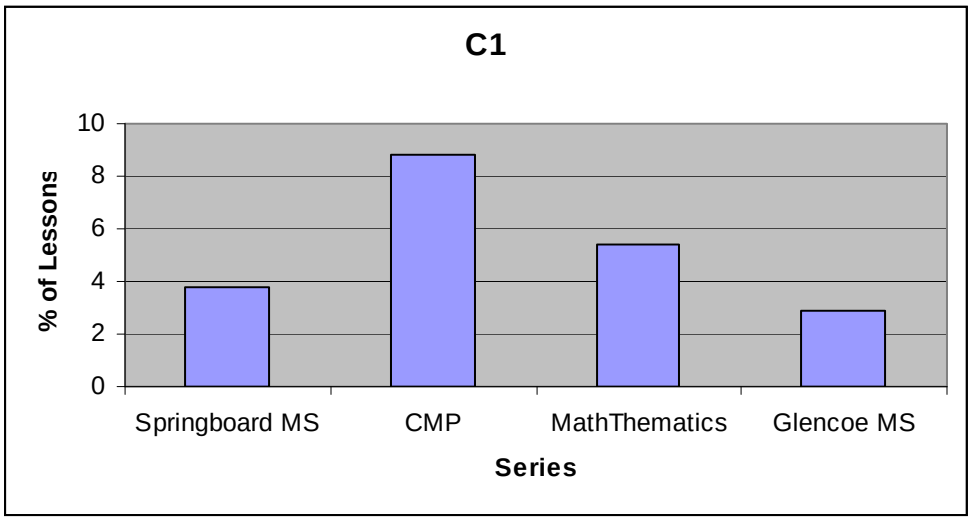
* NCTM % are approximate (Appendix C)
** Maine Learning Results Balance of Representation (Appendix D)

A breakdown of the Data Analysis and Probability alignment follows with the lesson alignment for the following four goal areas:

- Select and use appropriate statistical methods to analyze data
- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

Each of the Data Analysis and Probability Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success.

Data Analysis and Probability Goal Area1: Select and use appropriate statistical methods to analyze data (NCTM Goal Area Labeled C1 for this Analysis)



C1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

C: Data Analysis & Statistics

C1. Organize and analyze data using mean, median, mode, and range.

College Board Standards for College Success

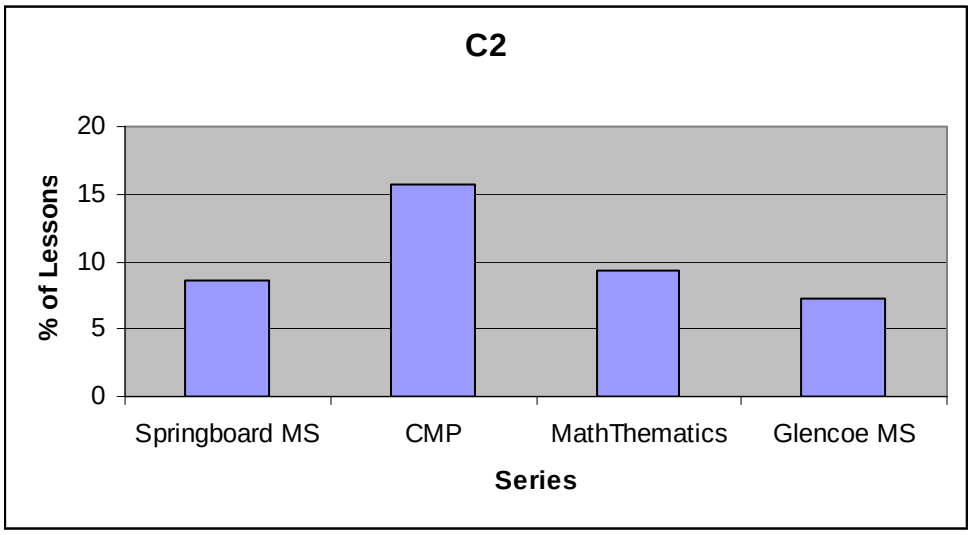
Standard: Data Analysis and Probability

Strand: Data Exploration

Level 1 Performance Expectation 1, 2, 4

Level 2 Performance Expectation 2, 3

Data Analysis and Probability Goal Area 2: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them (NCTM Goal Area Labeled C2 for this Analysis)



C2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

C: Data Analysis & Statistics

C2. Assemble data and use matrices to formulate and solve problems.

College Board Standards for College Success

Standard: Data Analysis and Probability

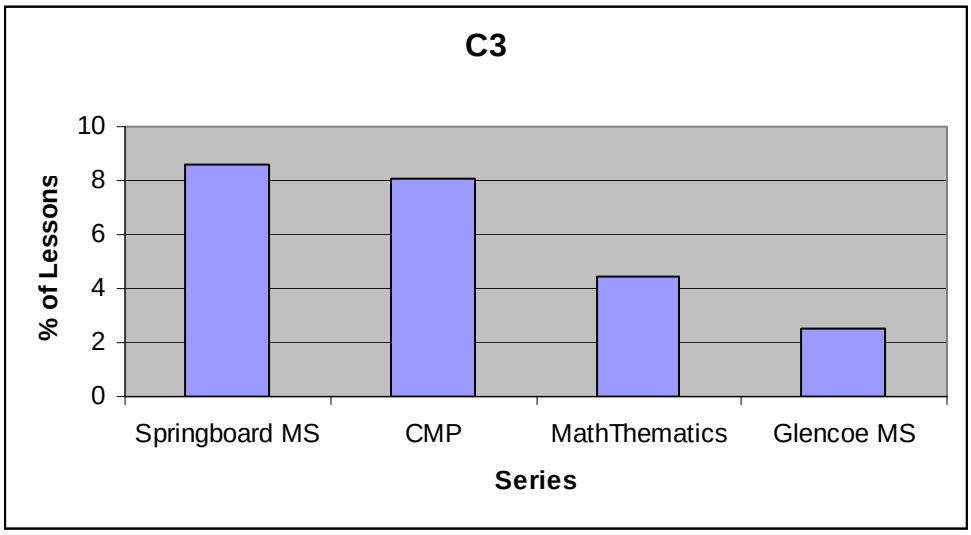
Strand: Collection

Level 1 Performance Expectation 1-3

Level 2 Performance Expectation 1-4

Level 3 Performance Expectation 1-2

Data Analysis and Probability Goal Area 3: Understand and apply basic concepts of probability (NCTM Goal Area Labeled C3 for this Analysis)



C3 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

D: Probability

D1. Find the probability of simple events and make predictions by applying the theories of probability.
D2. Explain the idea that probability can be represented as a fraction between and including zero and one.
D3. Use simulations to estimate probabilities.
D4. Find all possible combinations and arrangements involving a limited number of variables.

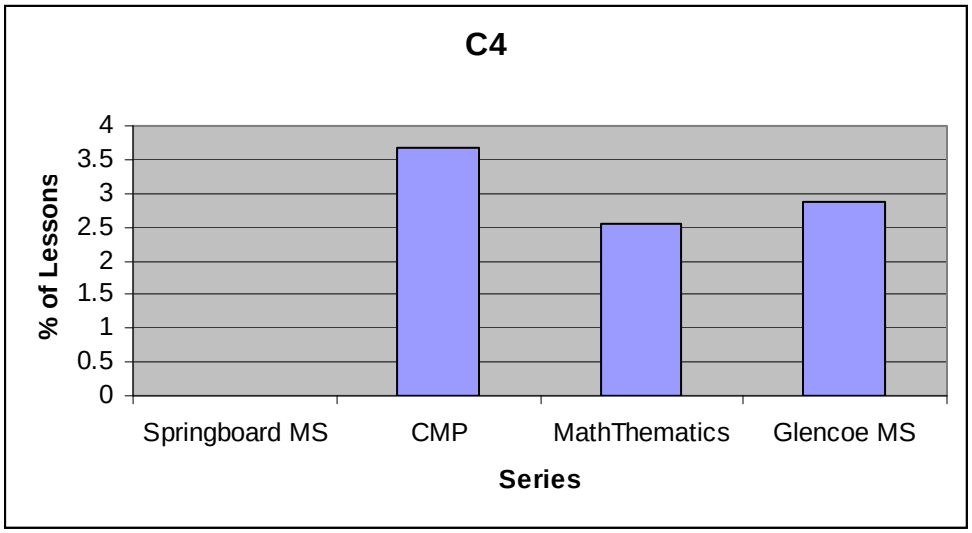
College Board Standards for College Success

Standard: Data Analysis and Probability

Strand: Probability Concepts and Applications

Level 1 Performance Expectation 1-5
Level 2 Performance Expectation 1-4
Level 3 Performance Expectation 1-31-2,4-5??

Data Analysis and Probability Goal Area 3: Develop and evaluate inferences and predictions that are based on data (NCTM Goal Area Labeled C4 for this Analysis)



C4 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

C: Data Analysis & Statistics

C3. Construct inferences and convincing arguments based on data.

College Board Standards for College Success

Standard: Data Analysis and Probability

Strand: Data Interpretation

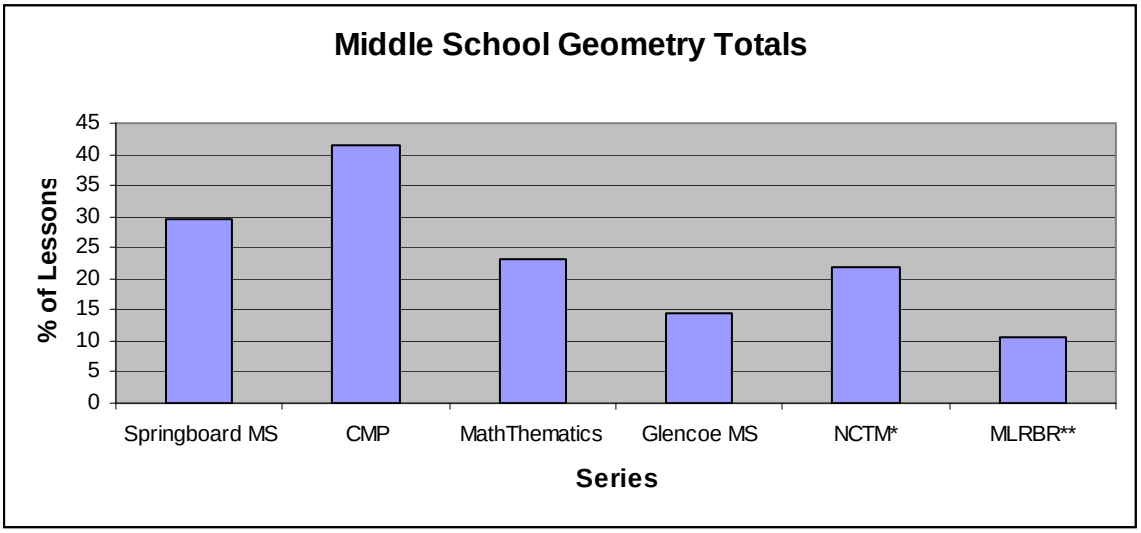
Level 1 Performance Expectation 1-2

Level 2 Performance Expectation 1-3

Level 3 Performance Expectation 1-3

Geometry

The NCTM content standards and the Maine Learning Results include the strand of Geometry. The above chart shows the percent of lessons aligned to the Geometry Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



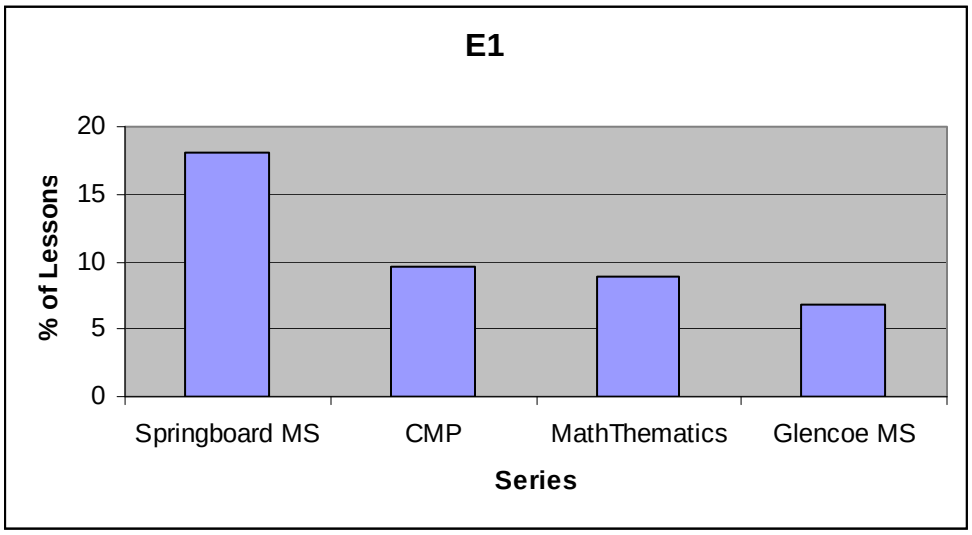
*NCTM % are approximate
**Maine Learning Results Balance of Representation

A breakdown of the Geometry alignment follows with the lesson alignment for the following four goal areas:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

Each of the Geometry Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success.

Geometry Goal Area 1: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships (NCTM Goal Area Labeled E1 for this Analysis)



E1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

E Geometry

E1. Compare, classify, and draw two dimensional shapes and three dimensional figures.

College Board Standards for College Success

Standard: Geometry

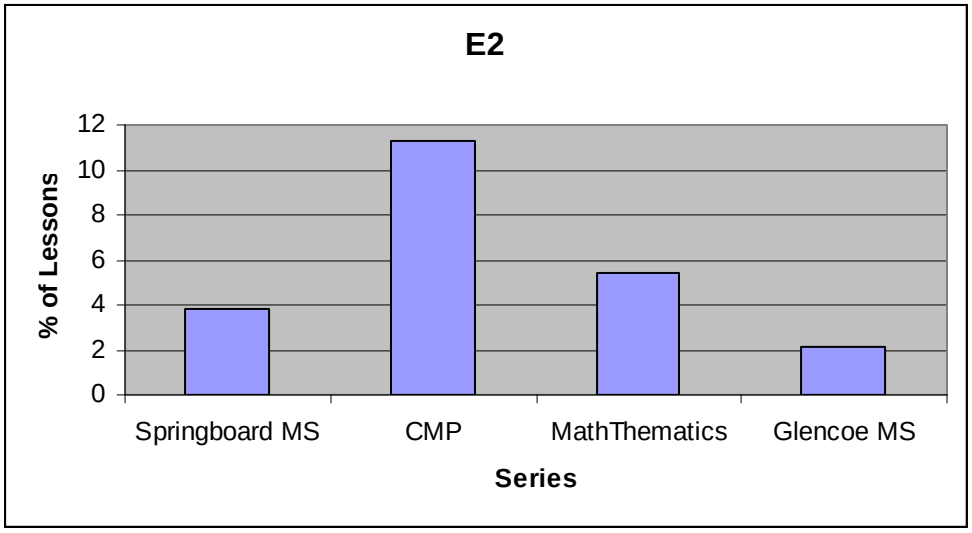
Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 1 Performance Expectation 2

Level 2 Performance Expectation 1-2

Level 3 Performance Expectation 1-2

Geometry Goal Area 2: Specify locations and describe spatial relationships using coordinate geometry and other representational systems (NCTM Goal Area Labeled E2 for this Analysis)



E2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

E Geometry

E3. Use a coordinate system to define and locate position.

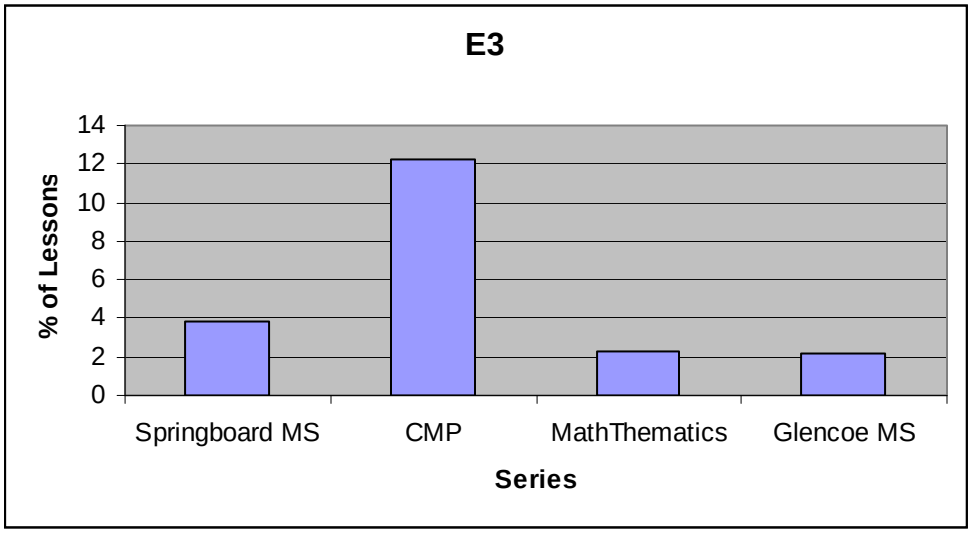
College Board Standards for College Success

Standard: Geometry

Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 3 Performance Expectation 3

Geometry Goal Area 3: Apply transformations and use symmetry to analyze mathematical situations (NCTM Goal Area Labeled E3 for this Analysis)



E3 Alignment to Maine Learning Results and College Board Standards

Maine Learning Results: Grades 5-8

E Geometry

E2. Apply geometric properties to represent and solve real-life problems involving regular and irregular shapes.

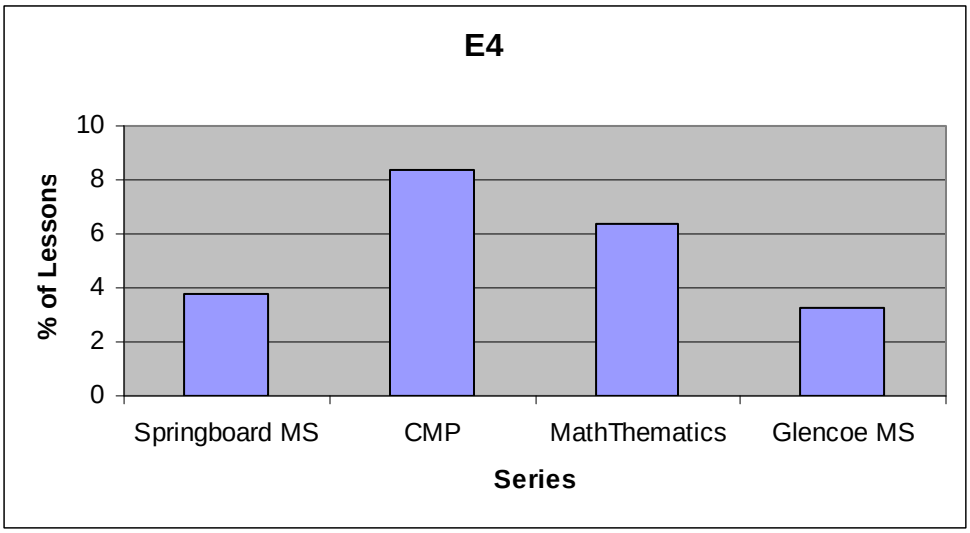
College Board Standards for College Success

Standard: Geometry

Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 3 Performance Expectation 1-2

Geometry Goal Area 4: Use visualization, spatial reasoning, and geometric modeling to solve problems (NCTM Goal Area Labeled E4 for this Analysis)



E4 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

E Geometry

E4. Use the appropriate geometric tools and measurements to draw and construct two and three-dimensional figures.

College Board Standards for College Success

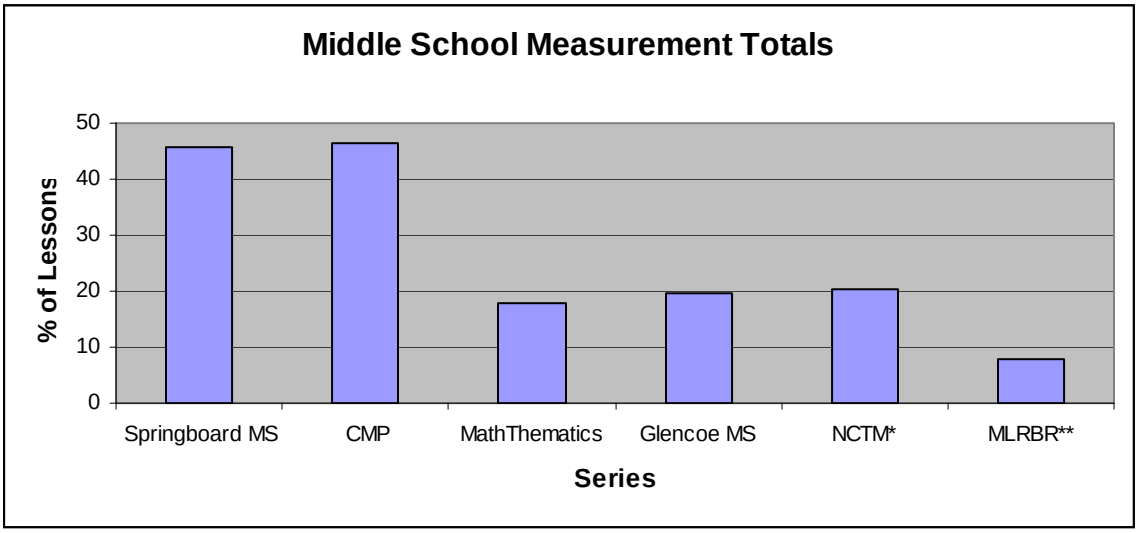
Standard: Geometry

Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 1 Performance Expectation 1

Measurement

Both the NCTM content standards and the Maine Learning Results Standards include a Measurement Strand. The above chart shows the percent of lessons aligned to the Measurement Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



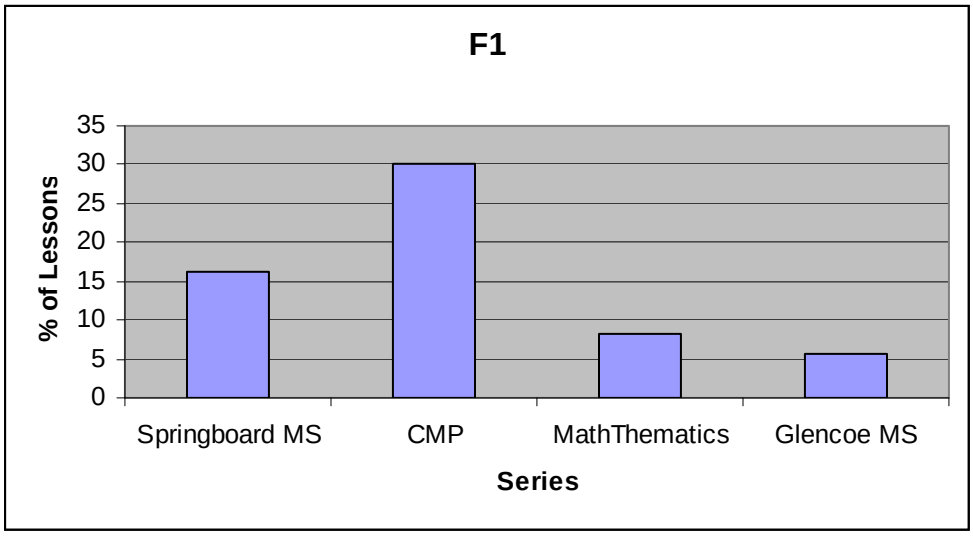
*NCTM % are approximate (Appendix C)
**Maine Learning Results Balance of Representation (Appendix D)

A breakdown of the Measurement Standard follows with the lesson alignment for the following two goal areas:

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

Each of the Measurement Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success.

Measurement Goal Area1: Understand measurable attributes of objects and the units, systems, and processes of measurement (NCTM Goal Area Labeled F1 for this Analysis)



F1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

F: Measurement

F1. Demonstrate the structure and use of systems of measurement.

College Board Standards for College Success

Standard: Measurement

Strand: Measurement Unit and Systems

Level 1 Performance Expectation 1-2

Level 2 Performance Expectation 1-2

Level 3 Performance Expectation 1-2

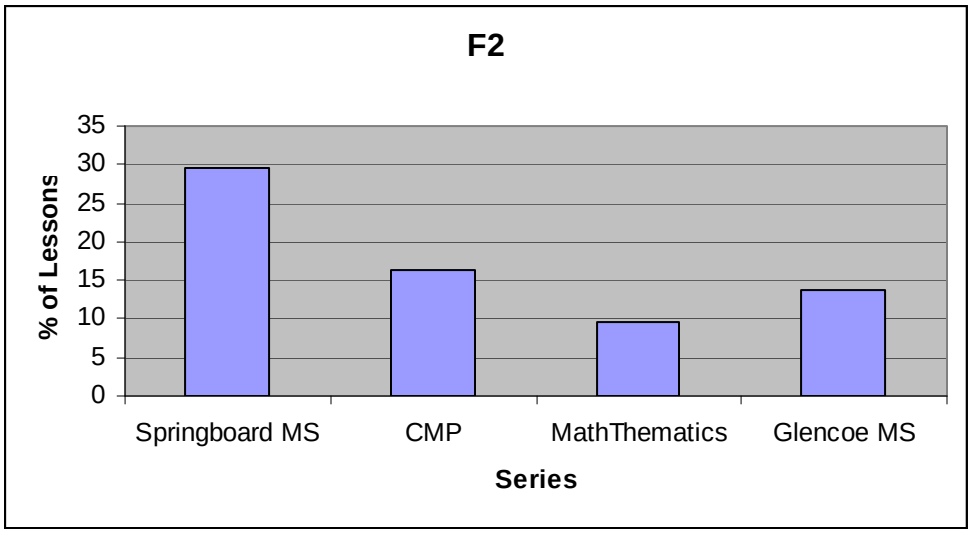
Strand: Conversion with Measures

Level 1 Performance Expectation 1-2

Level 2 Performance Expectation 1-2

Level 3 Performance Expectation 1-3

Measurement Goal Area 2: Apply appropriate techniques, tools, and formulas to determine measurements (NCTM Goal Area Labeled F2 for this Analysis)



F2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

F: Measurement

F2. Develop and use concepts that can be measured directly, or indirectly (e.g., the concept of rate).

F3. Demonstrate an understanding of length, area, volume, and the corresponding units, square units, and cubic units of measure.

College Board Standards for College Success

Standard: Measurement

Strand: Applications of Measurement

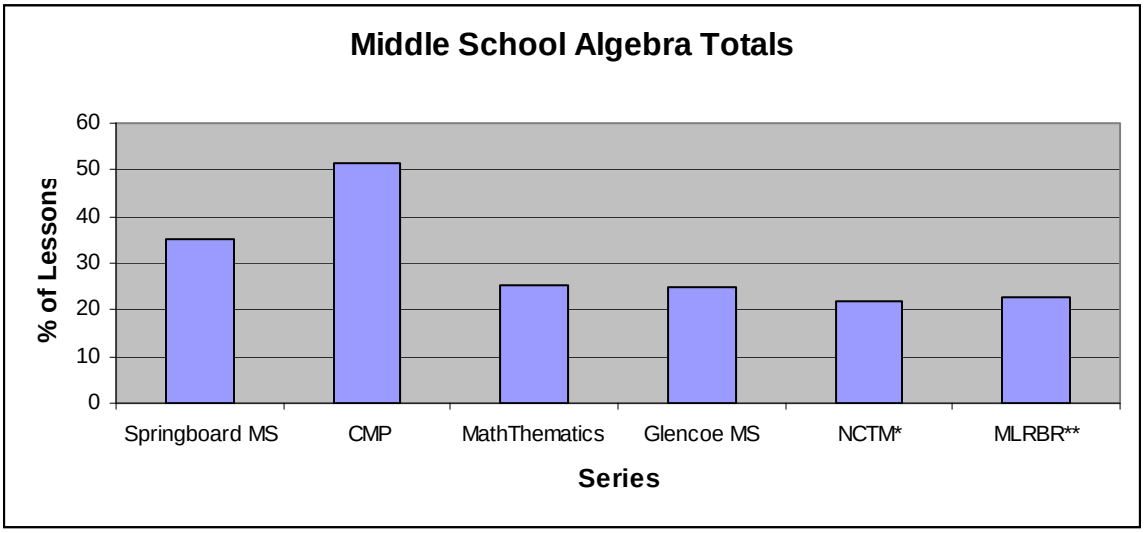
Level 1 Performance Expectation 1-3

Level 2 Performance Expectation 1-2

Level 3 Performance Expectation 1-2

Algebra

The NCTM content standard, Algebra, encompasses two of the Maine Learning Results Standards, Algebra and Patterns, Relations & Functions. The above chart shows the percent of lessons aligned to the Algebra Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



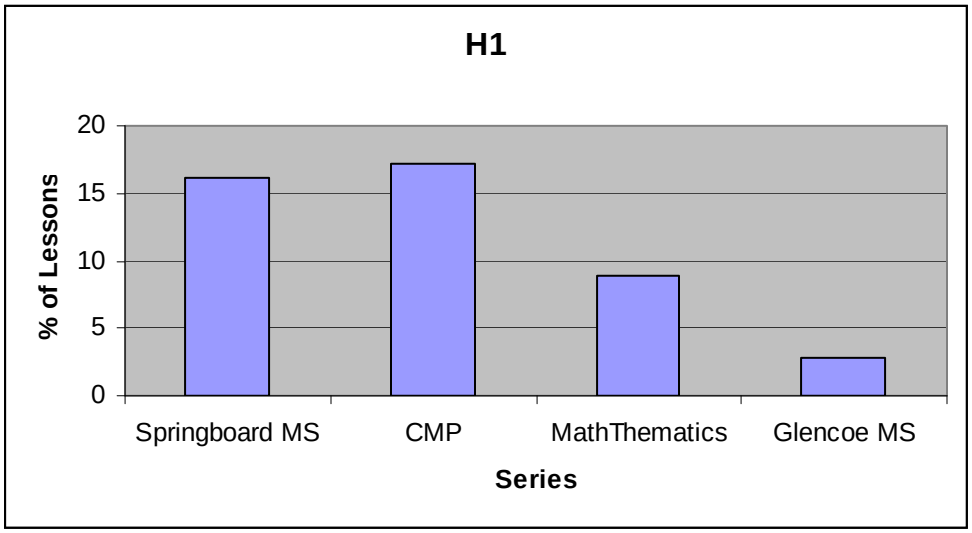
*NCTM % are approximate (Appendix C)
**Maine Learning Results Balance of Representation (Appendix D)

A breakdown of the Algebra alignment follows with the lesson alignment for the following four goal areas:

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

Each of the Algebra Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success. There is much overlap in the Algebra and Patterns, Relations, Functions performance indicators of the Maine Learning Results

Algebra Goal Area 1: Understand patterns, relations, and functions (NCTM Goal Area Labeled H1 for this Analysis)



H1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

G: Patterns, Relations, and Functions

G1. Describe and represent relationships with tables, graphs, and equations.

G3. Use patterns and multiple representations to solve problems

H: Algebra

H4. Use graphs to represent two-variable equations.

H5. Demonstrate an understanding of inequalities and non-linear equations.

College Board Standards for College Success

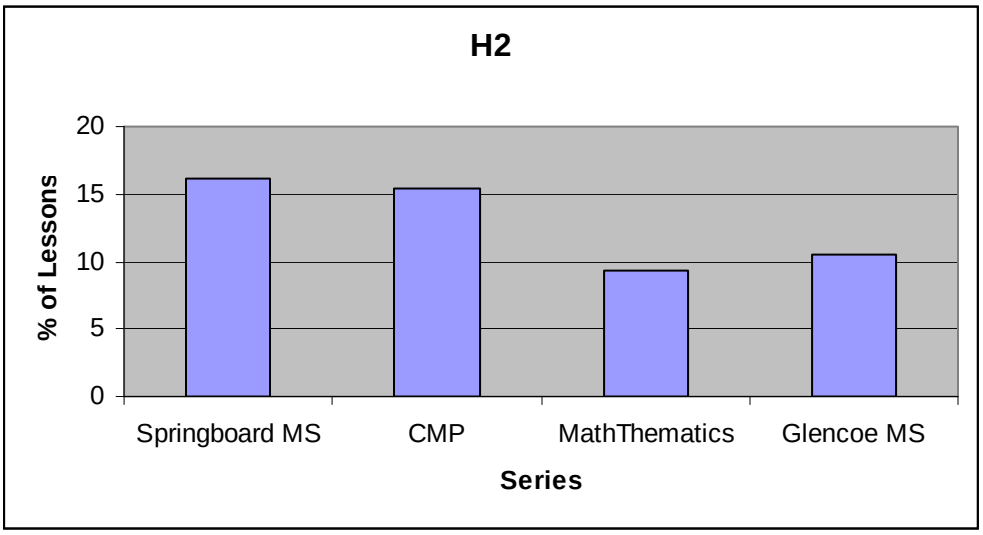
Standard: Algebra

Strand: Patterns and Relations

Level 1 Performance Expectation 1-3

Level 2 Performance Expectation 1-2

Algebra Goal Area 2: Represent and analyze mathematical situations and structures using algebraic symbols (NCTM Goal Area Labeled H2 for this Analysis)



H2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

H: Algebra

H1. Use the concepts of variables and expressions.

H2. Solve linear equations using concrete, informal, and formal methods which apply the order of operations.

H5. Demonstrate an understanding of inequalities and non-linear equations.

H6. Find solutions for unknown quantities in linear equations and in simple equations and inequalities.

College Board Standards for College Success

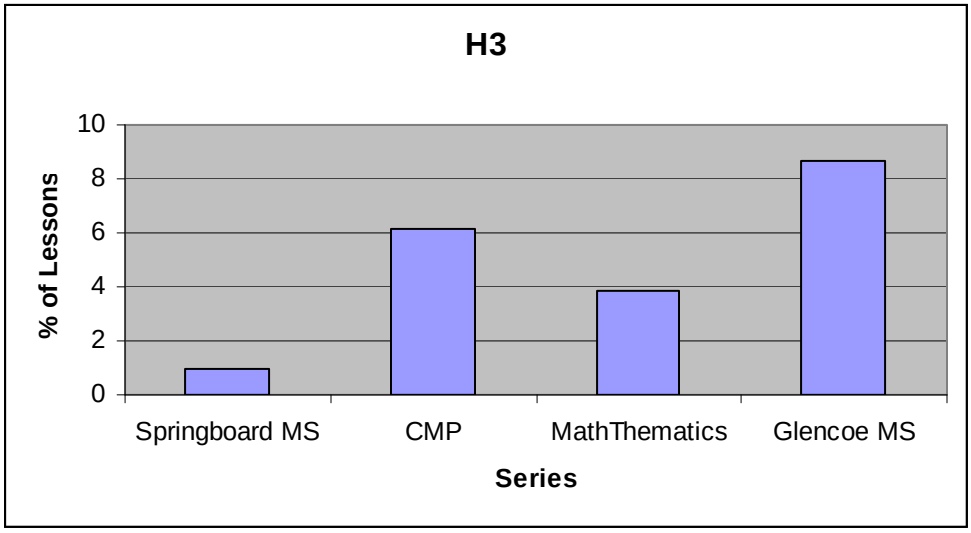
Standard: Algebra

Strand: Variables, Expressions, Equations, and Inequalities

Level 1 Performance Expectation 1-5

Level 2 Performance Expectation 1-5

Algebra Goal Area H3: Use mathematical models to represent and understand quantitative relationships (NCTM Goal Area Labeled H3 for this Analysis)



H3 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

G: Patterns, Relations, Functions

G3. Use patterns and multiple representations to solve problems

H: Algebra

H3. Analyze tables and graphs to identify properties and relationships in a practical context.

College Board Standards for College Success

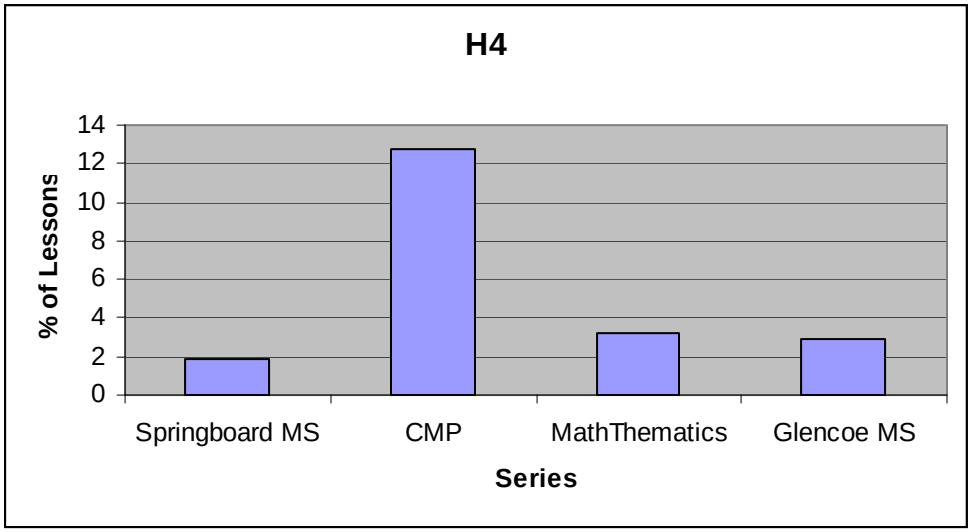
Standard: Algebra

Strand: Variables, Expressions, Equations, and Inequalities

Level 1 Performance Expectation 3

Level 2 Performance Expectation 1

Algebra Goal Area H4: Analyze change in various contexts (NCTM Goal Area Labeled H4 for this Analysis)



H4 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 5-8

G: Patterns, Relations, Functions

G1. Describe and represent relationships with tables, graphs, and equations.

G2. Analyze relationships to explain how a change in one quantity can result in a change in another.

H: Algebra

H3. Analyze tables and graphs to identify properties and relationships in a practical context.

College Board Standards for College Success

Standard: Algebra

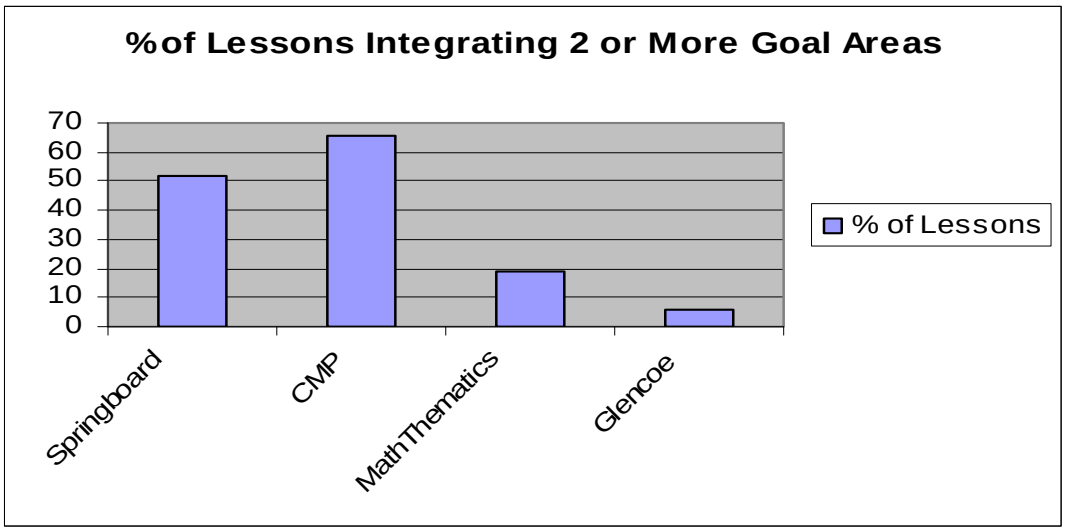
Strand: Patterns and Relations

Level 2 Performance Expectation 2

Summary of Middle Level Comparison

The five content standards of NCTM's National Standards for School Mathematics are represented in each of the programs analyzed for this report. With the exception of Glencoe's Geometry coverage and Math Thematics' Measurement coverage, all other program areas were above the balance recommended by NCTM (see Appendix C) In comparing the percent of lessons focusing on a goal area to the DOE's Balance of Representation (see Appendix D), all 4 programs were at or above the determined balance.

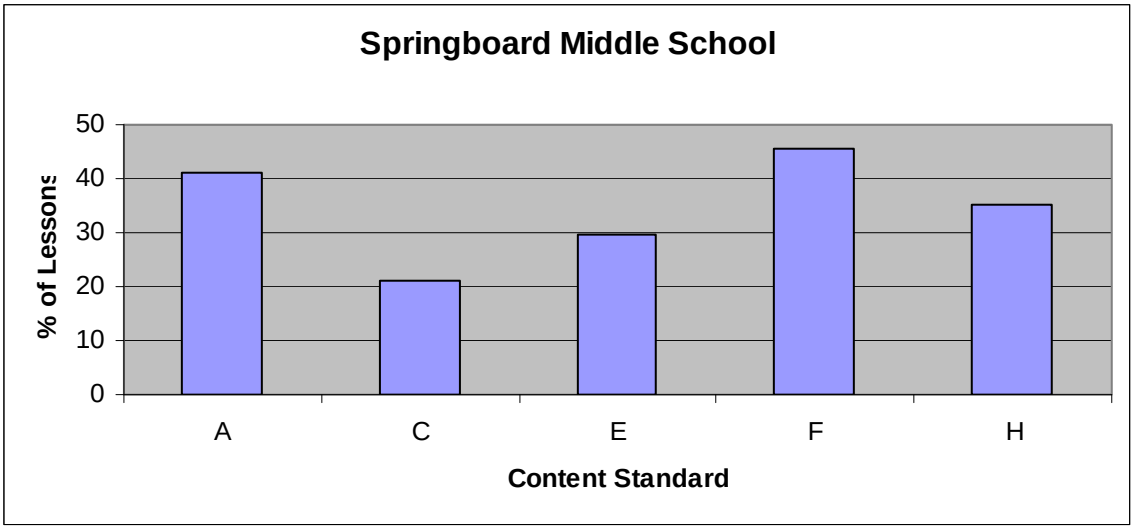
In many areas, CMP and Springboard show a greater percentage than either the NCTM or Balance of Representation. This results from the programs' integration of goal areas. As shown by the graph below, both CMP and Springboard have a higher number of lessons that incorporate 2 or more of the goal areas. In both programs, this is accomplished by using a problem centered approach to mathematics.



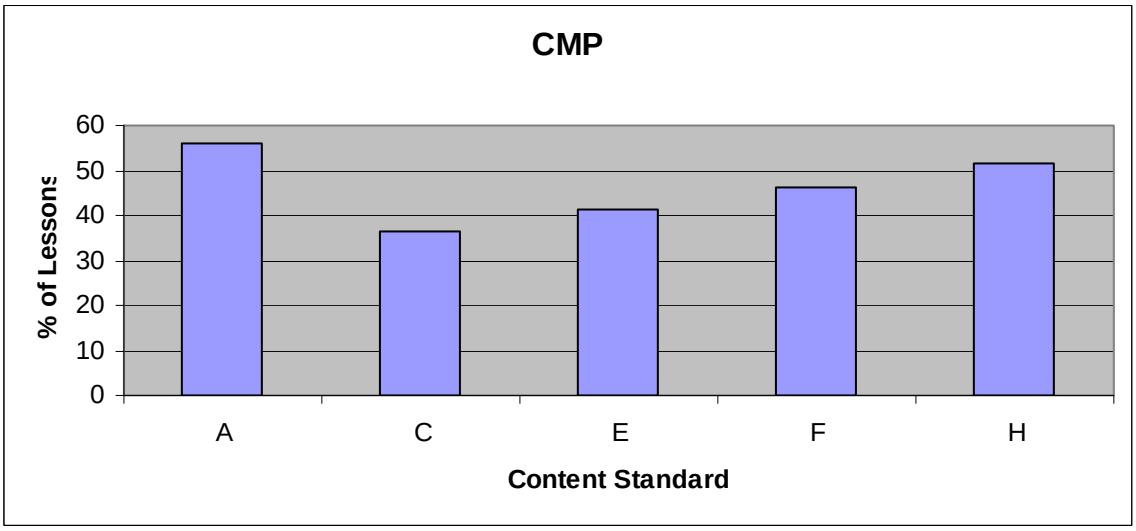
MathThematics also uses a problem solving approach but the integration of goal areas is within the unit rather than individual lessons.

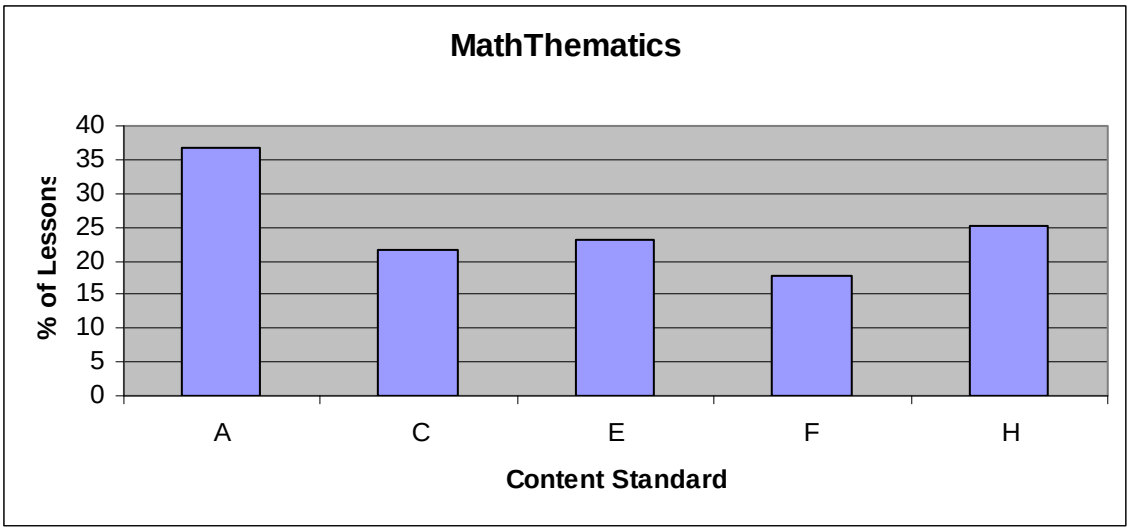
Content Totals By Program

The charts that follow give a picture of the content coverage for each of the programs analyzed.

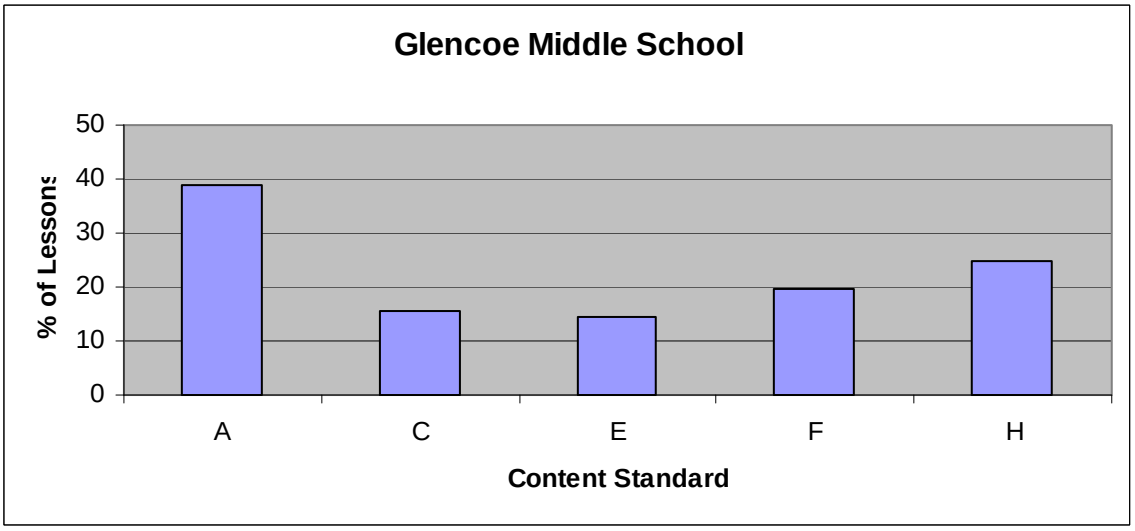


A: Numbers and Operations
C: Data Analysis & Probability
E: Geometry
F: Measurement
H: Algebra

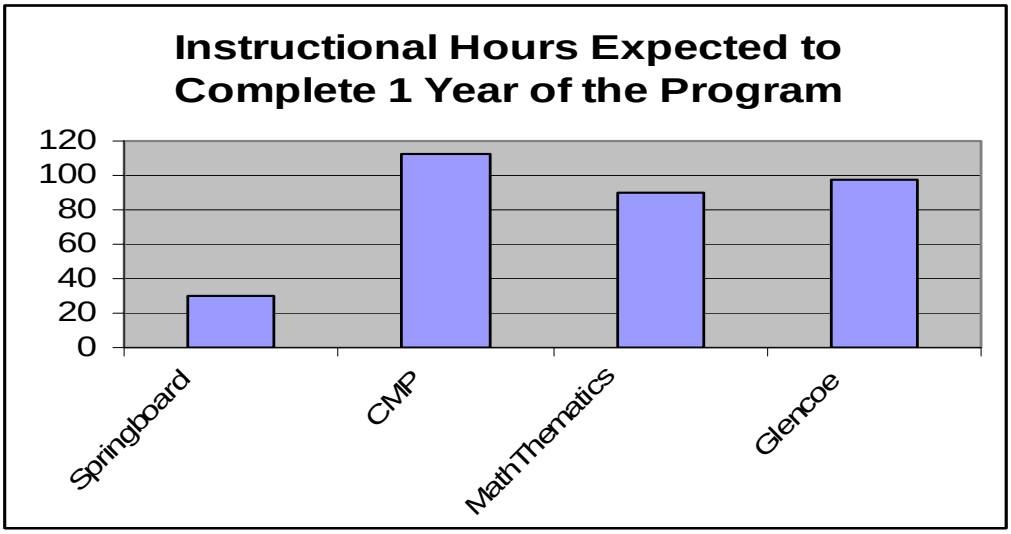




A: Numbers and Operations
C: Data Analysis & Probability
E: Geometry
F: Measurement
H: Algebra



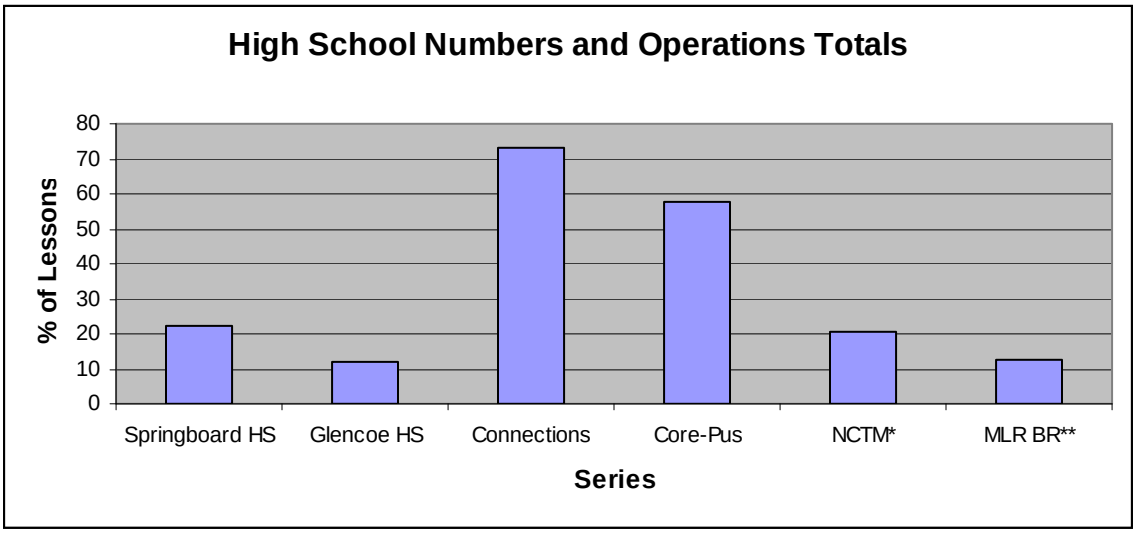
Another important consideration when analyzing a program's content coverage is to look at time expectations for covering the material. The following chart summarizes the expected number of hours for completing one year of the program. The expectation is derived from the written guidelines for coverage of each lesson/activity. Actual time required will vary due to a number of contextual factors including student prior knowledge and classroom discourse.



High School Comparison

Numbers and Operations (NCTM Standard)

The NCTM content standard, Numbers and Operations, encompasses two of the Maine Learning Results Standards, Number Sense and Computation. The above chart shows the percent of lessons aligned to the Numbers and Operations Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



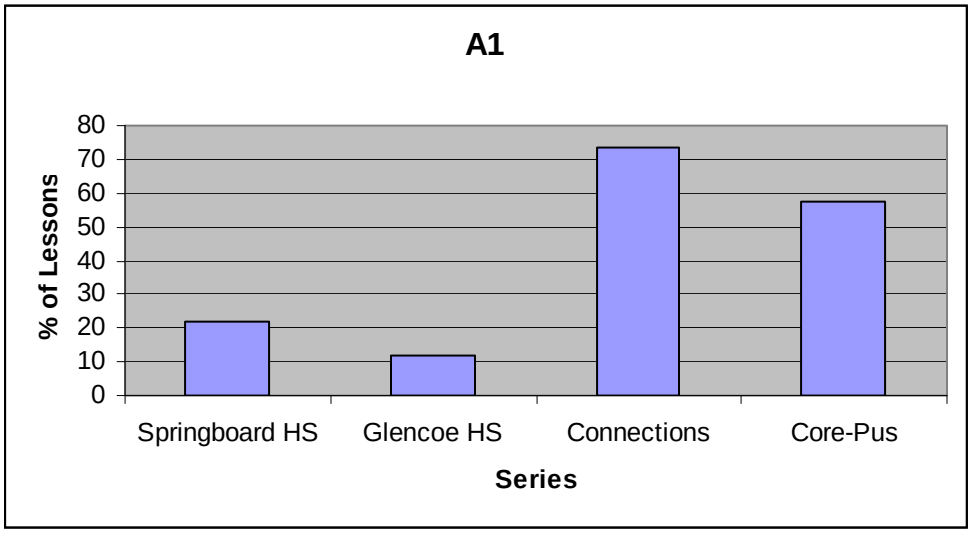
*NCTM % are approximate (Appendix C)
**Maine Learning Results Balance of Representation (Appendix D)

A breakdown of the Numbers and Operations alignment follows with the lesson alignment for the following three goal areas:

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates

Each of the Number and Operations Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success

Numbers and Operations Goal Area 1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems (NCTM Goal Area Labeled A1 for this Analysis)



A1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

A. Numbers and Numbers Sense

A1. Describe the structure of the real number system and identify its appropriate applications and limitations.

A2. Explain what complex numbers (real and imaginary) mean and describe some of their many uses.

College Board Standards for College Success

Standard: Numbers and Operations

Strand: Number

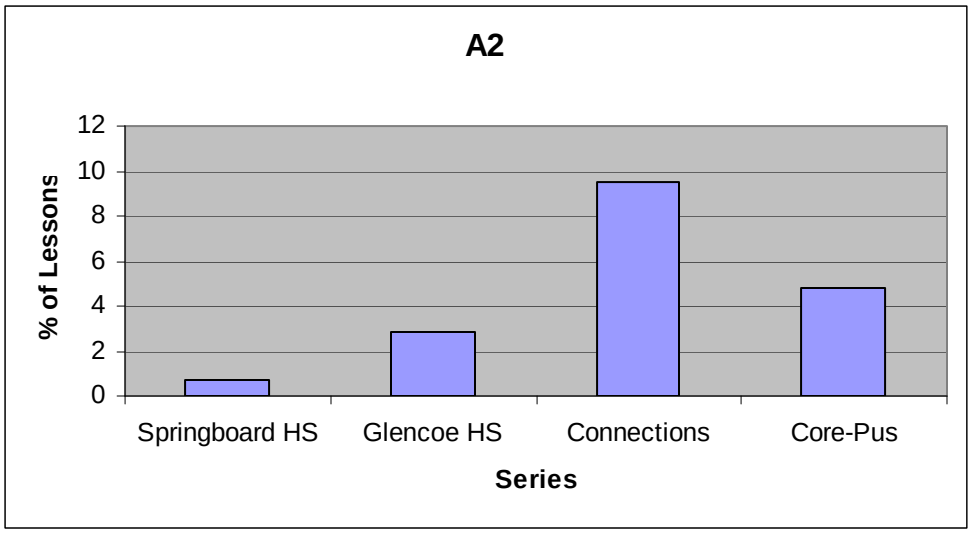
Level 4 Performance Expectation 1-2

Level 5 Performance Expectation 1-2

Strand: Computation and Estimation

Level 5 Performance Expectations 4-5

Numbers and Operations Goal 2: Understand meanings of operations and how they relate to one another (NCTM Goal Area Labeled A2 for this Analysis)



A2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

B. Computation

B1. Use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.

B2. Explain operations with number systems other than base ten.

College Board Standards for College Success

Standard: Numbers and Operations

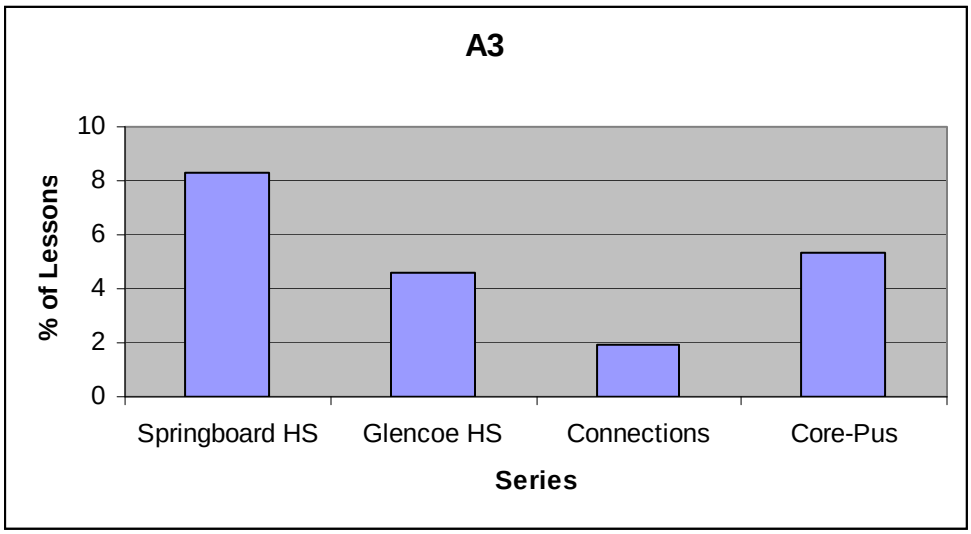
Strand: Operations

Level 3 Performance Expectation 1

Level 4 Performance Expectation 1-4

Level 5 Performance Expectation 1-3

Numbers and Operations Goal 3: Compute fluently and make reasonable estimates (NCTM Goal Area Labeled A3 for this Analysis)



A3 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

B. Computation

B1. Use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.

B2. Explain operations with number systems other than base ten.

College Board Standards for College Success

Standard: Numbers and Operations

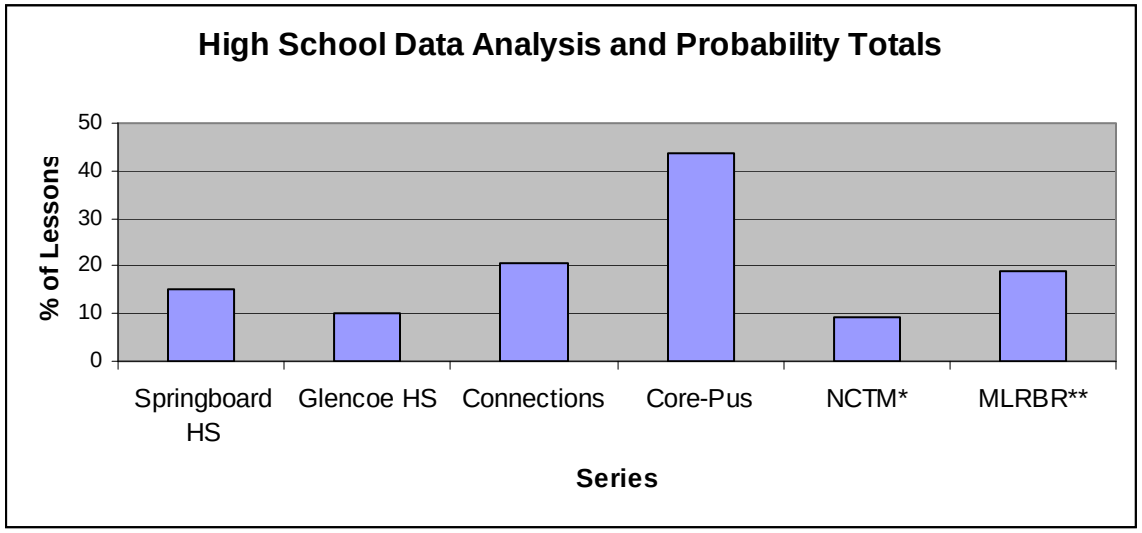
Strand: Computation and Estimation

Level 4 Performance Expectation 1-3

Level 5 Performance Expectation 1-4

Data Analysis and Probability

The NCTM content standard, Data Analysis and Probability, encompasses two of the Maine Learning Results Standards, Data Analysis& Statistics and Probability. The above chart shows the percent of lessons aligned to the Data Analysis and Probability Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



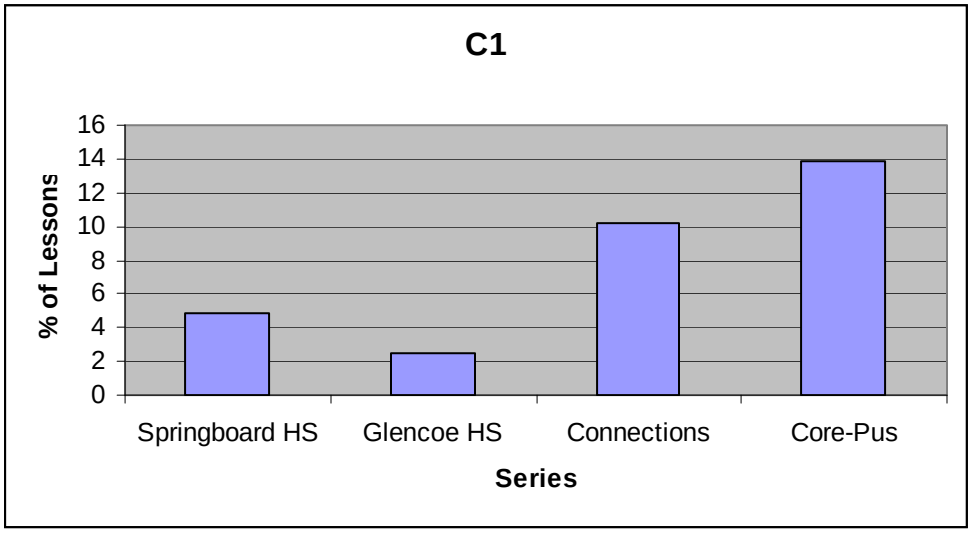
*NCTM % are approximate
**Maine Learning Results Balance of Representation

A breakdown of the Data Analysis and Probability alignment follows with the lesson alignment for the following four goal areas:

- Select and use appropriate statistical methods to analyze data
- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

Each of the Data Analysis and Probability Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success.

Data Analysis and Probability Goal Area 1: Select and use appropriate statistical methods to analyze data (NCTM Goal Area Labeled C1 for this Analysis)



C1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

C: Data Analysis & Statistics

C3. Demonstrate an understanding of concepts of standard deviation and correlation and how they relate to data analysis.

College Board Standards for College Success

Standard: Data Analysis and Probability

Strand: Data Exploration

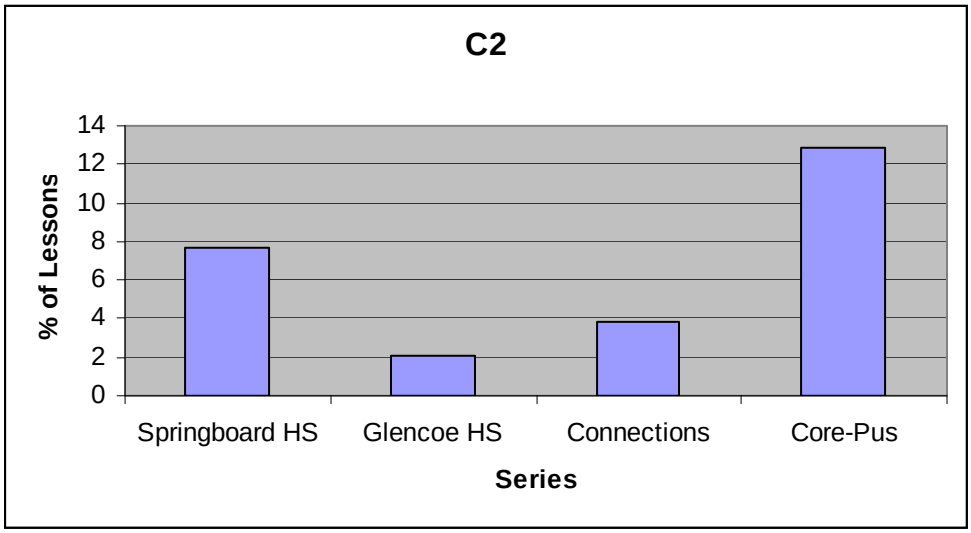
Level 4 Performance Expectation 1,2

Level 5 Performance Expectation 1-4

Strand: Data Interpretation

Level 5 Performance Expectation 2

Data Analysis and Probability Goal Area 2: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them (NCTM Goal Area Labeled C2 for this Analysis)



C2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

C: Data Analysis & Statistics

C2. Predict and draw conclusions from charts, tables, and graphs that summarize data from practical situations.

C4. Demonstrate an understanding of the idea of random sampling and recognition of its role in statistical claims and designs for data collection.

College Board Standards for College Success

Standard: Data Analysis and Probability

Strand: Collection

Level 4 Performance Expectation 2,4

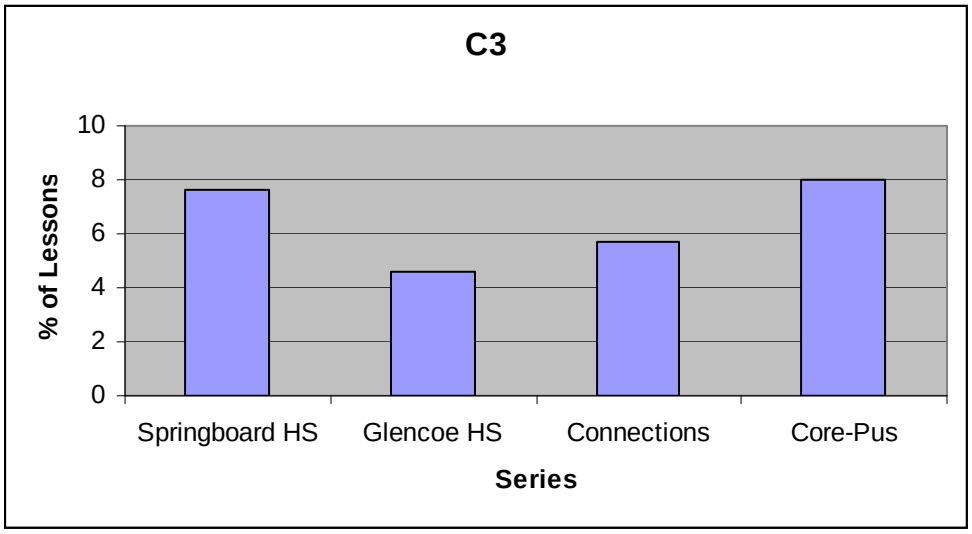
Level 5 Performance Expectation 1,2,4

Strand: Data Exploration

Level 4 Performance Expectation 3,4

Level 5 Performance Expectation 1,2

Data Analysis and Probability Goal Area 3: Understand and apply basic concepts of probability (NCTM Goal Area Labeled C4 for this Analysis)



C3 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

D: Probability

D1. Find the probability of compound events and make predictions by applying probability theory.

D2. Create and interpret probability distributions.

College Board Standards for College Success

Standard: Data Analysis and Probability

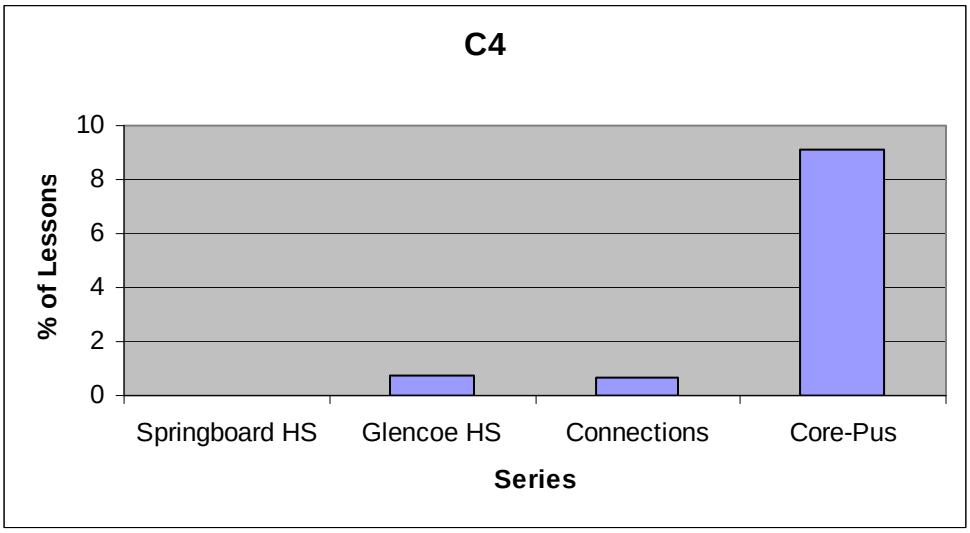
Strand: Probability Concepts and Applications

Level 3 Performance Expectation 1,2,4

Level 4 Performance Expectation 1-5

Level 5 Performance Expectation 1-4

Data Analysis and Probability Goal Area 4: Develop and evaluate inferences and predictions that are based on data (NCTM Goal Area Labeled C3 for this Analysis)



C4 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

C: Data Analysis & Statistics

C1. Determine and evaluate the effect of variables on the results of data collection.

C3. Construct inferences and convincing arguments based on data.

C5. Revise studies to improve their validity (e.g., in terms of better sampling, better controls, or better data analysis techniques).

College Board Standards for College Success

Standard: Data Analysis and Probability

Strand: Data Exploration

Level 4 Performance Expectation 3

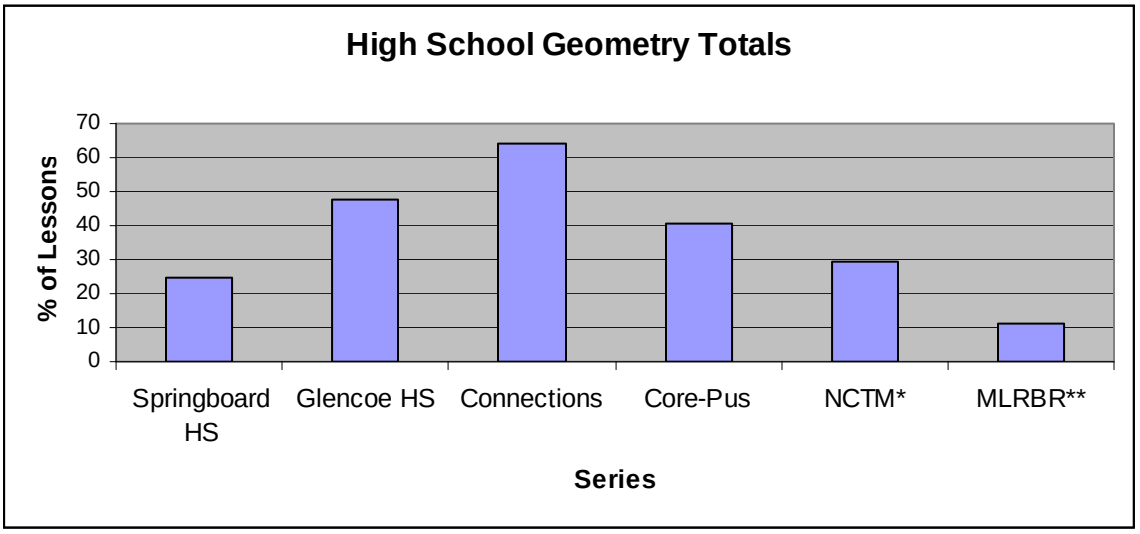
Strand: Data Interpretation

Level 4 Performance Expectation 1-4

Level 5 Performance Expectation 1,3

Geometry

The NCTM content standards and the Maine Learning Results include the strand of Geometry. The above chart shows the percent of lessons aligned to the Geometry Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



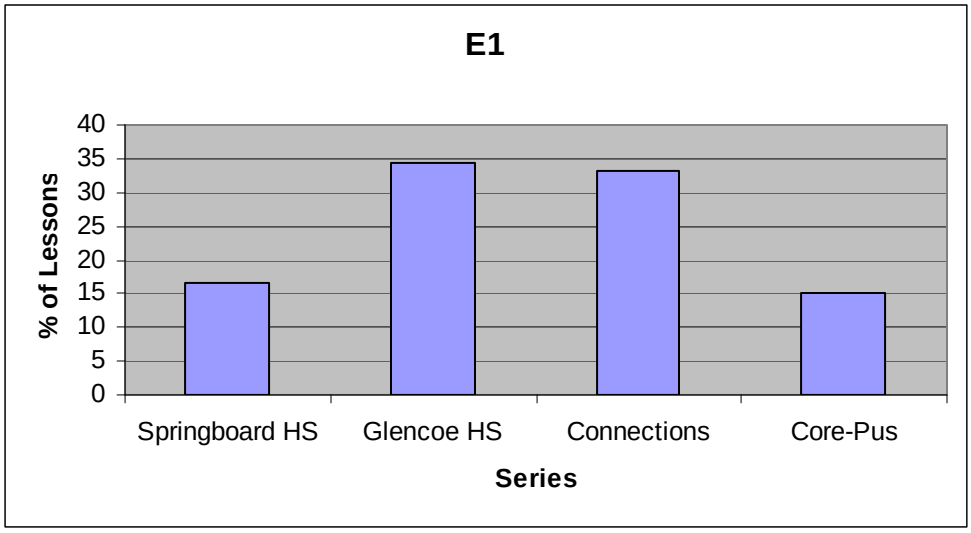
*NCTM % are approximate
**Maine Learning Results Balance of Representation

A breakdown of the Geometry alignment follows with the lesson alignment for the following four goal areas:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

Each of the Geometry Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success.

Geometry Goal Area 1: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships (NCTM Goal Area Labeled E1 for this Analysis)



E1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

E Geometry

E2. Use inductive and deductive reasoning to explore and determine the properties of and relationships among geometric figures.

E3. Apply trigonometry to problem situations involving triangles and periodic phenomena.

College Board Standards for College Success

Standard: Geometry

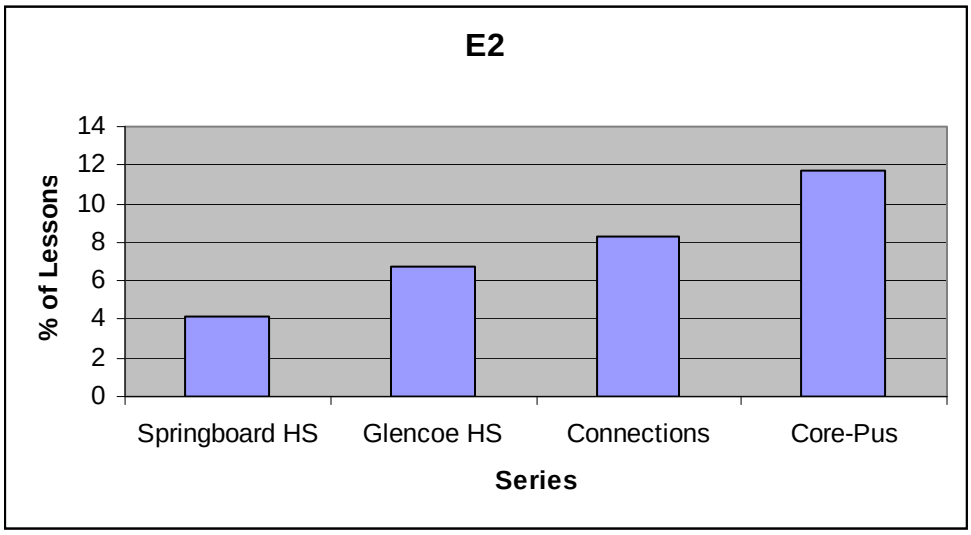
Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 3 Performance Expectation 2

Level 4 Performance Expectation 1-2

Level 5 Performance Expectations 1-2

Geometry Goal Area 2: Specify locations and describe spatial relationships using coordinate geometry and other representational systems (NCTM Goal Area Labeled E2 for this Analysis)



E2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

E Geometry

E1. Draw coordinate representations of geometric figures and their transformations.

College Board Standards for College Success

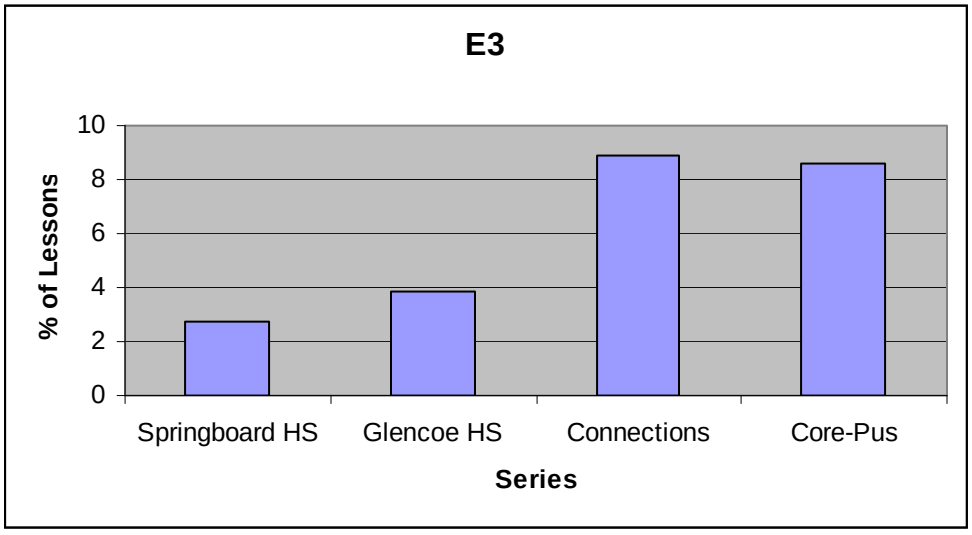
Standard: Geometry

Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 4 Performance Expectation 3

Level 5 Performance Expectation 4

Geometry Goal Area 3: Apply transformations and use symmetry to analyze mathematical situations (NCTM Goal Area Labeled E3 for this Analysis)



E3 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

E Geometry

E1. Draw coordinate representations of geometric figures and their transformations.

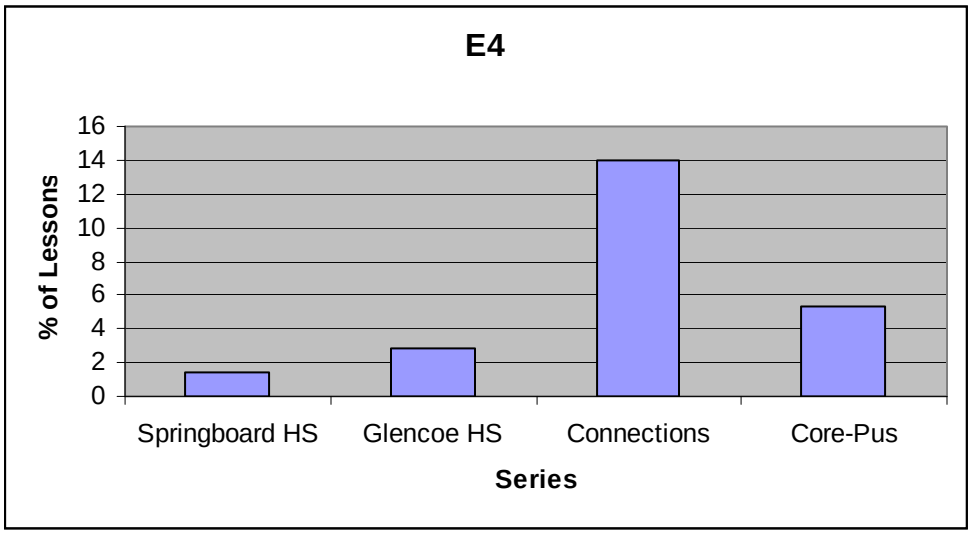
College Board Standards for College Success

Standard: Geometry

Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 3 Performance Expectation 2

Geometry Goal Area 4: Use visualization, spatial reasoning, and geometric modeling to solve problems (NCTM Goal Area Labeled E4 for this Analysis)



E4 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results

No indicator alignment

College Board Standards for College Success

Standard: Geometry

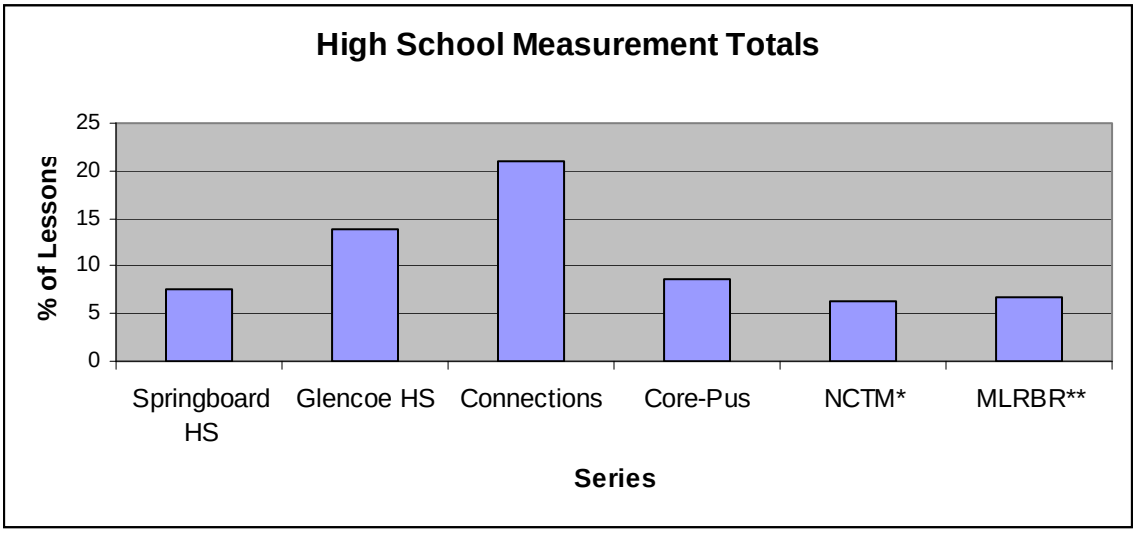
Strand: Properties, Attributes, and Models of Plane and Solid Figures

Level 4 Performance Expectation 1-2

Level 5 Performance Expectation 1,4

Measurement

Both the NCTM content standards and the Maine Learning Results Standards include a Measurement Strand. The above chart shows the percent of lessons aligned to the Measurement Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



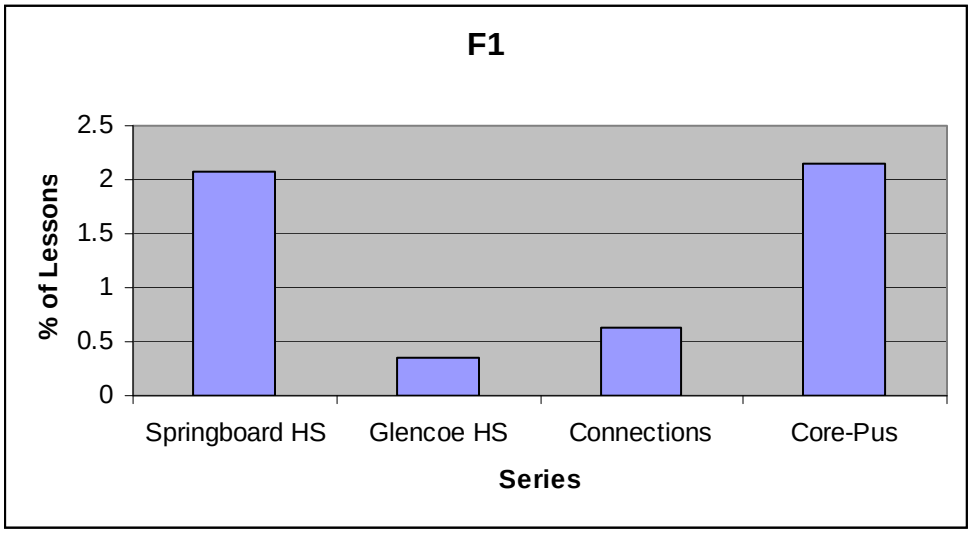
*NCTM % are approximate (Appendix C)
**Maine Learning Results Balance of Representation (Appendix D)

A breakdown of the Measurement Standard follows with the lesson alignment for the following two goal areas:

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

Each of the Measurement Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success.

Measurement Goal Area 1: Understand measurable attributes of objects and the units, systems, and processes of measurement (NCTM Goal Area Labeled F1 for this Analysis)



F1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

F: Measurement

F1. Use measurement tools and units appropriately and recognize limitations in the precision of the measurement tools

College Board Standards for College Success

Standard: Measurement

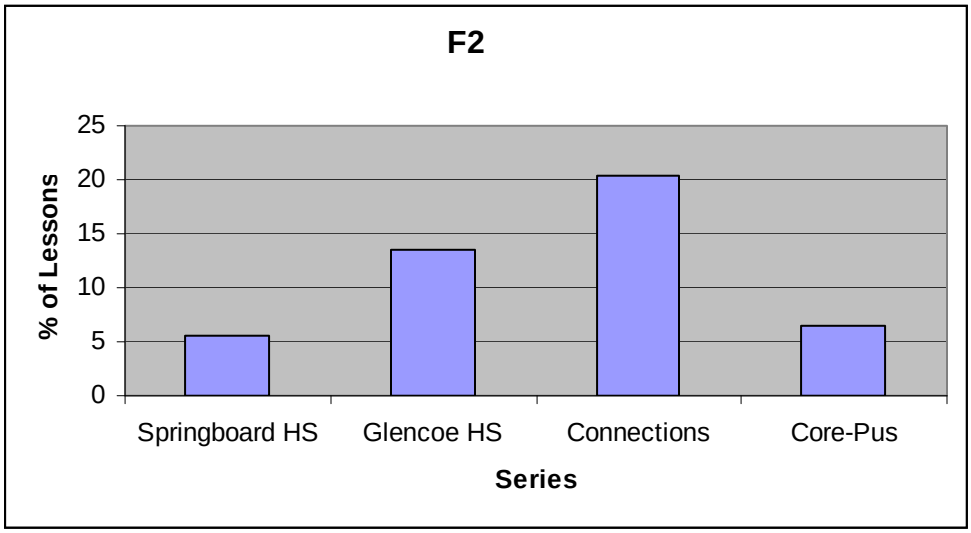
Strand: Measurement Unit and Systems

Level 4 Performance Expectation 1-2

Strand: Applications of Measurement

Level 5 Performance Expectation 1

Measurement Goal Area 2: Apply appropriate techniques, tools, and formulas to determine measurements (NCTM Goal Area for this Analysis)



F2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

F: Measurement

F1. Use measurement tools and units appropriately and recognize limitations in the precision of the measurement tools.

F2. Derive and use formulas for area, surface area, and volume of many types of figures.

College Board Standards for College Success

Standard: Measurement

Strand: Applications of Measurement

Level 4 Performance Expectation 1-2

Level 5 Performance Expectation 1-3

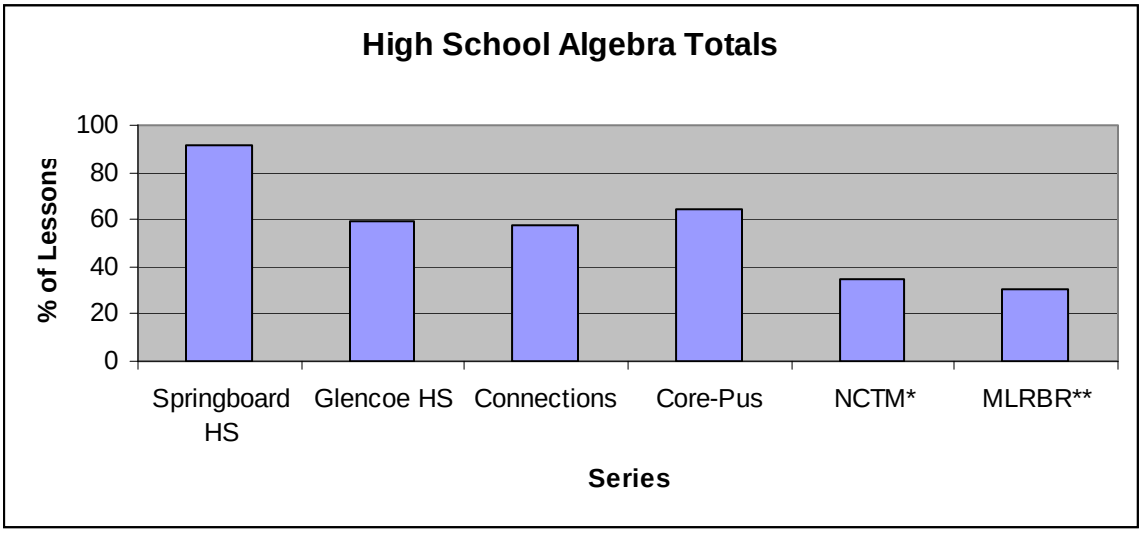
Strand: Conversion with Measures

Level 4 Performance Expectation 1-2

Level 5 Performance Expectation 1-2

Algebra

The NCTM content standard, Algebra, encompasses two of the Maine Learning Results Standards, Algebra and Patterns, Relations & Functions. The above chart shows the percent of lessons aligned to the Algebra Standard and the recommended instructional balance from the Principles and Standards and from the ME Department of Education's Balance of Representation.



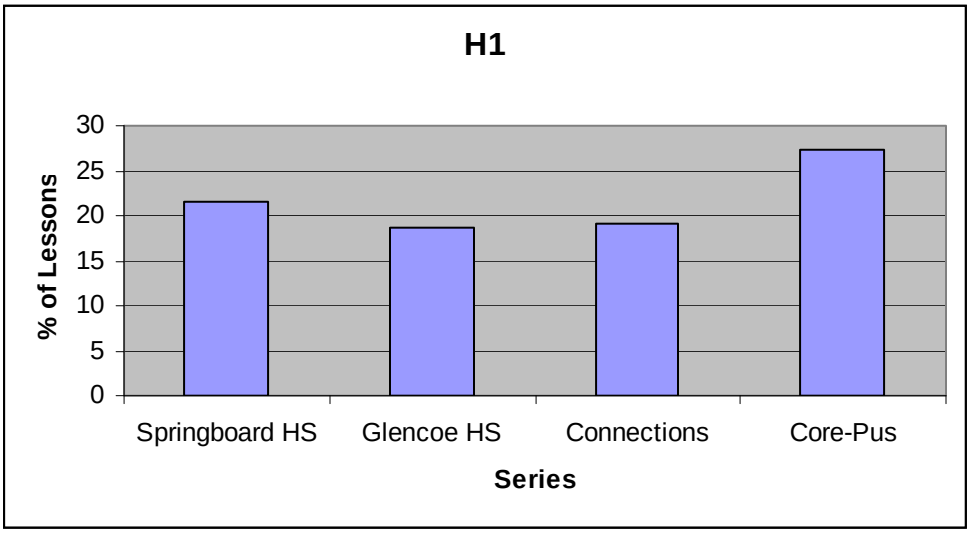
*NCTM % are approximate
**Maine Learning Results Balance of Representation

A breakdown of the Algebra alignment follows with the lesson alignment for the following four goal areas:

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

Each of the Algebra Goal Areas is aligned to the Maine Learning Results and the College Board Standards for College Success. There is much overlap in the Algebra and Patterns, Relations, Functions performance indicators of the Maine Learning Results

Algebra Goal Area 1: Understand patterns, relations, and functions (NCTM Goal Area Labeled H1 for this Analysis)



H1 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

G: Patterns, Relations, and Functions

G2. Translate and solve a real-life problem using symbolic language.

G4. Identify a variety of situations explained by the same type of function.

H: Algebra

H2. Investigate concepts of variation by using equations, graphs, and data collection.

College Board Standards for College Success

Standard: Algebra

Strand: Patterns and Relations

Level 3 Performance Expectation 1-3

Level 4 Performance Expectation 1-2

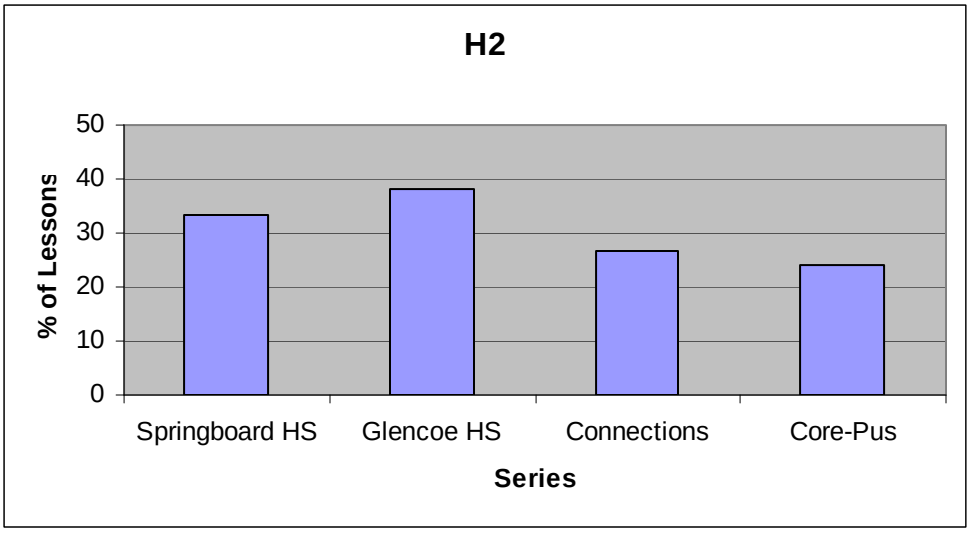
Level 5 Performance Expectation 1-3

Strand: Functions with Growth and Change

Level 4 Performance Expectation 2-5

Level 5 Performance Expectation 1-5

Algebra Goal Area 2: Represent and analyze mathematical situations and structures using algebraic symbols (NCTM Goal Area Labeled H2 for this Analysis)



H2 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

G: Patterns, Relations, Functions

G2. Translate and solve a real-life problem using symbolic language

H: Algebra

H1. Use tables, graphs, and spreadsheets to interpret expressions, equations, and inequalities.

H2. Investigate concepts of variation by using equations, graphs, and data collection.

H3. Formulate and solve equations and inequalities.

H4. Analyze and explain situations using symbolic representations.

College Board Standards for College Success

Standard: Algebra

Strand: Variables, Expressions, Equations, and Inequalities

Level 2 Performance Expectation 1-5

Level 3 Performance Expectation 1-5

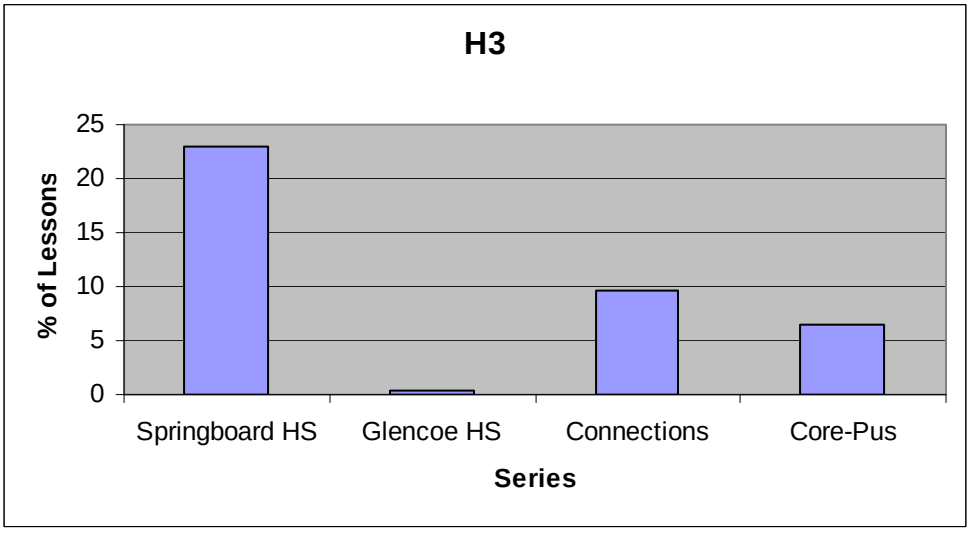
Level 4 Performance Expectation 1-5

Level 5 Performance Expectation 1-4

Strand: Functions with Growth and Change

Level 4 Performance Expectation 1

Algebra Goal Area 3: Use mathematical models to represent and understand quantitative relationships (NCTM Goal Area Labeled H3 for this Analysis)



H3 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

G: Patterns, Relations, Functions

G1. Create a graph to represent a real-life situation and draw inferences from it.

G3.Model phenomena using a variety of functions (linear, quadratic, exponential, trigonometric, etc.).

College Board Standards for College Success

Standard: Algebra

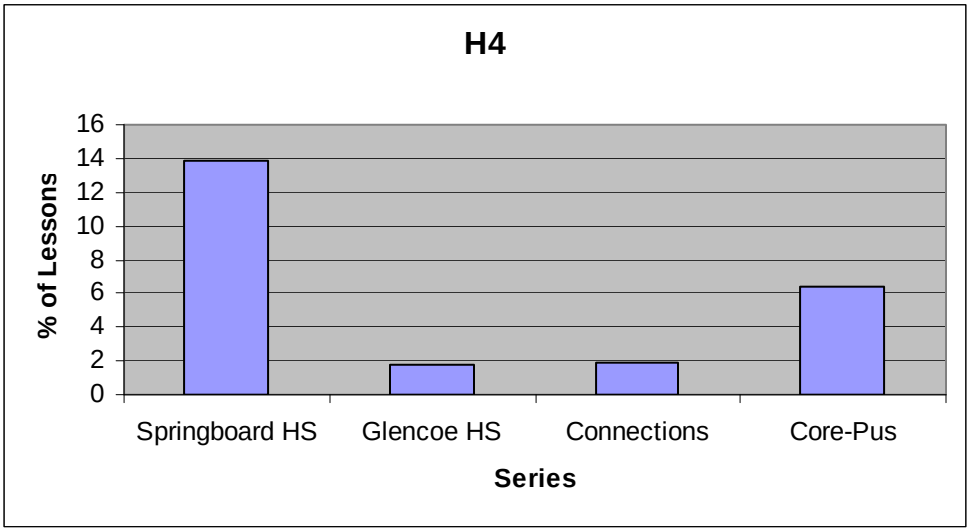
Strand: Variables, Expressions, Equations, and Inequalities

Level 3 Performance Expectation 1

Level 4 Performance Expectation 1

Level 5 Performance Expectation 1

Algebra Goal Area 4: Analyze change in various contexts (NCTM Goal Area Labeled H4 for this Analysis)



H4 Alignment to Maine Learning Results and College Board Standards:

Maine Learning Results: Grades 9-12

G: Patterns, Relations, Functions

G1. Create a graph to represent a real-life situation and draw inferences from it.

H: Algebra

H2. Investigate concepts of variation by using equations, graphs, and data collection.

College Board Standards for College Success

Standard: Algebra

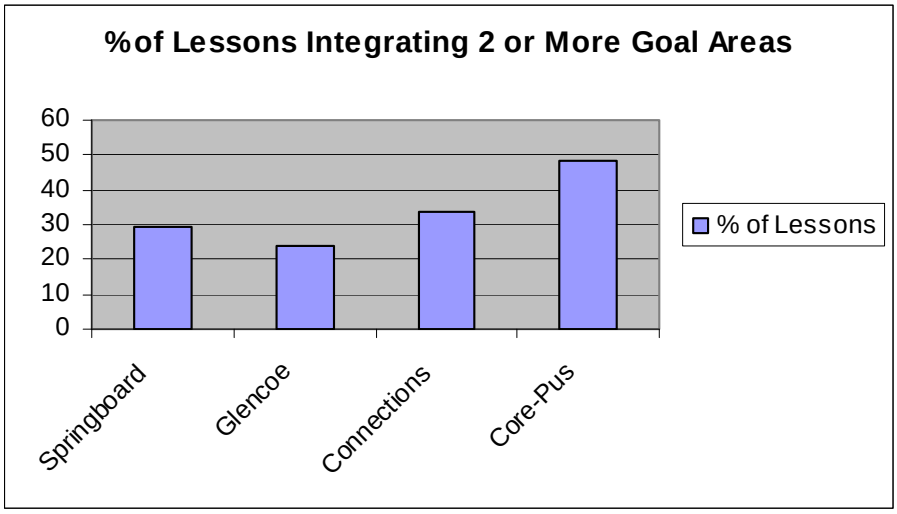
Strand: Functions with Growth and Change

Level 4 Performance Expectation 3-4

Summary of High School Content

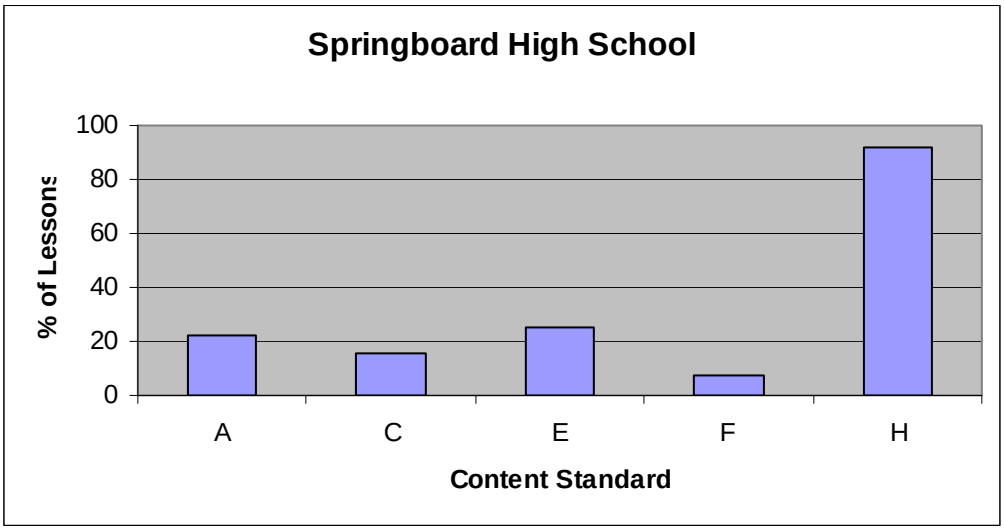
The five content standards of the National Standards are represented in each of the programs analyzed for this report. With the exception of Glencoe's Numbers & Operations coverage and SpingBoard's Geometry coverage, all other program areas were above the growth expectations recommended by NCTM (see Appendix C). In comparing the percent of lessons focusing on a goal area to the DOE's Balance of Representation (see Appendix D), all program areas were at or above the determined balance with the exception of Glencoe's and SpingBoard's Data Analysis & Probability coverage.

In many areas, Core-Plus, Connections and Springboard show a greater percentage than either the NCTM or Balance of Representation. This results from the programs' integration of goal areas. As shown by the graph below, the programs have a number of lessons that incorporate 2 or more of the goal areas. This is accomplished by using a problem centered approach to mathematics.

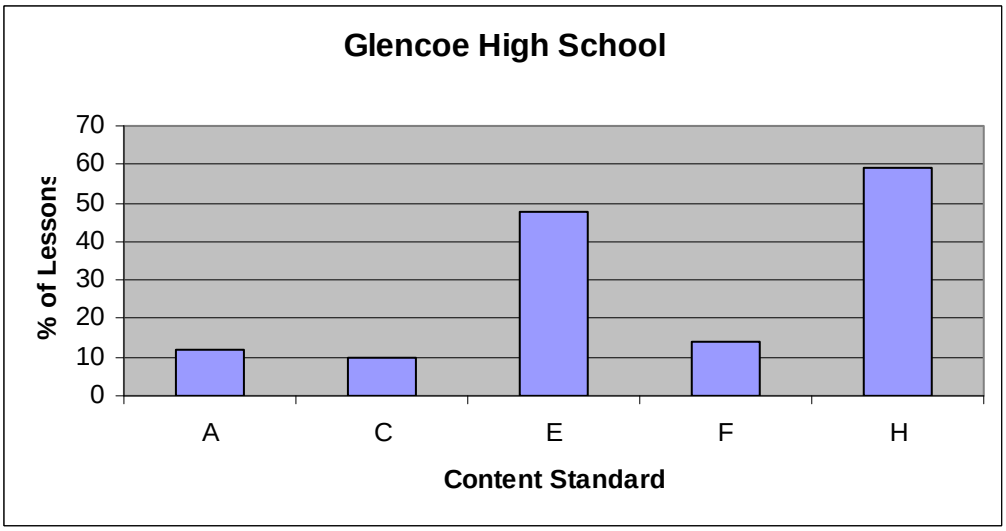


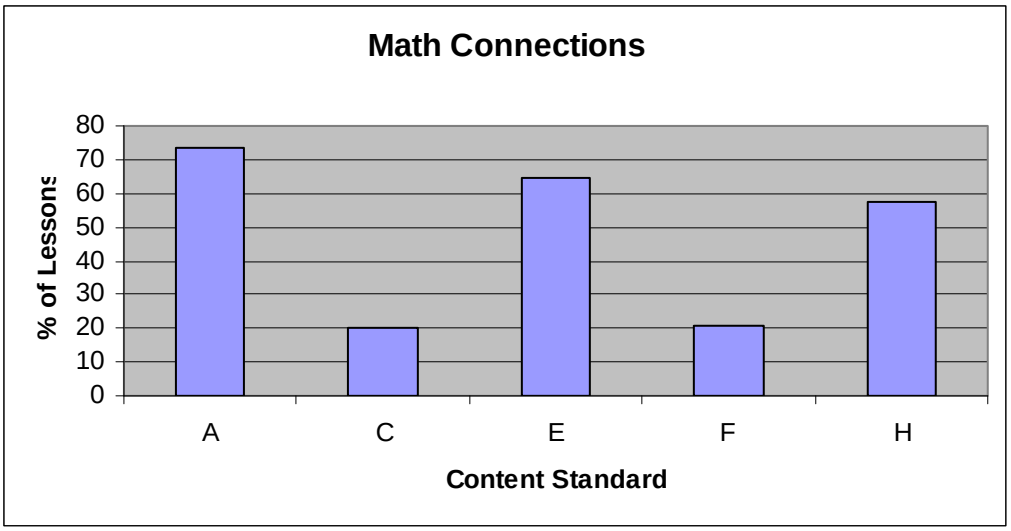
Content Overview by Program

The charts that follow give a picture of the content coverage for each of the programs analyzed.

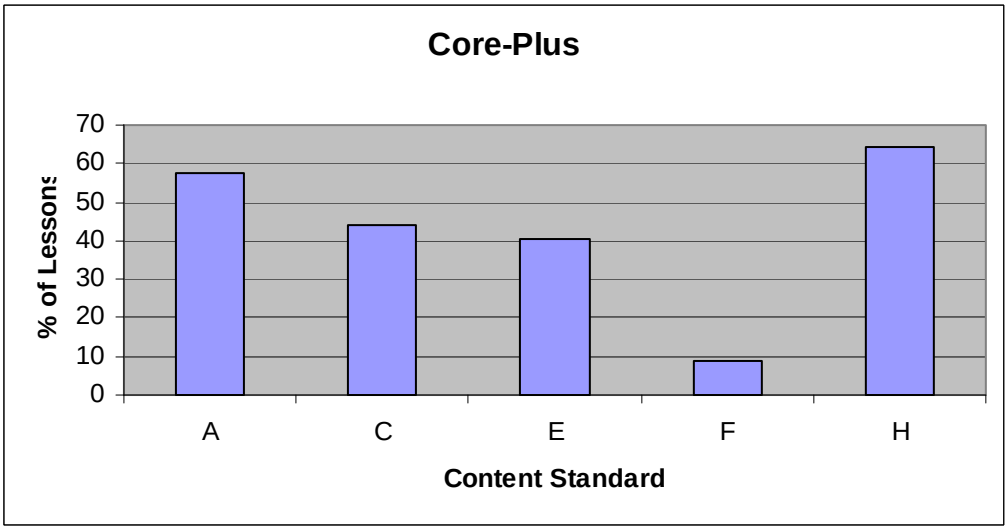


A: Numbers and Operations
C: Data Analysis & Probability
E: Geometry
F: Measurement
H: Algebra

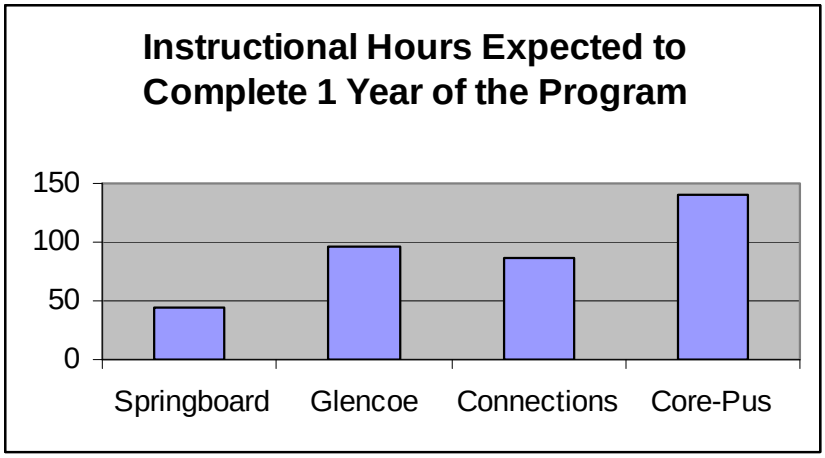




A: Numbers and Operations
C: Data Analysis & Probability
E: Geometry
F: Measurement
H: Algebra



Another important consideration when analyzing a program's content coverage is to look at time expectations for covering the material. The following chart summarizes the expected number of hours for completing one year of the program. The expectation is derived from the written guidelines for coverage of each lesson/activity. Actual time required will vary due to a number of contextual factors including student prior knowledge and classroom discourse.



VI. Comparison of Instructional Criteria

Analyzing the curriculum materials for alignment between instruction and the selected learning goal(s) of each activity involves estimating the degree to which the written components of the material reflect what is known generally about student learning and effective teaching and, more important, the degree to which they support student learning of the specific knowledge and skills for which a content match has been found (Project 2061, 2000). Six of the areas from Project 2061's procedure were used in this analysis. Each of the Categories are broken into 2-6 criteria which is further detailed using indicators. The Categories were used for the program analysis with the indicators serving as criteria for a low, moderate or high alignment rating (see Appendix B for list of Categories, Criteria, and Indicators). The overall rating of each of the categories below is provided as the typical alignment for each of the programs. Since the alignment does vary depending on the goal area, type of activity, and order within a unit, the overall rating reflects the majority of the ratings.

Category I: Identifying a Sense of Purpose
I.1 Conveying Unit Purpose: Does the lesson/activity convey an overall sense of purpose and direction that is understandable and motivating to students?

I.2 Conveying Lesson Purpose: Does the lesson/activity convey the purpose of each activity or lesson and its relationship to others

I.3 Justifying Sequence of Activities: Does the lesson/activity involve students in a logical or strategic sequence of activities (versus a collection of activities) that build toward understanding of the ideas in the unit or chapter purpose?

Overall Ratings for Category I

Middle Level Program	I.1	I.2	I.3
SpringBoard	Low/Moderate	Moderate	Moderate
CMP	Moderate/High	Moderate/High	Moderate/High
MathThematics	Moderate	Moderate	Low/Moderate
Glencoe	Moderate	Moderate	Moderate
High School Program	I.1	I.2	I.3
SpringBoard	Low/Moderate	Moderate	Moderate
Glencoe	Moderate	Low	Low/Moderate
Math Connections	Moderate	Moderate	Moderate
Core-Plus	Moderate	Moderate/High	Moderate

Category II: Building on Student Ideas about Mathematics

II.1 Specifying Prerequisite Knowledge. Does the lesson/activity specify and address prerequisite knowledge/skills that are necessary to the learning of the learning goal?

II.2 Alerting Teacher to Student Ideas. Does the lesson/activity alert teachers to commonly held student ideas (both troublesome and helpful) such as those described in *Learning goals for Science Literacy* Chapter 15: The Research Base?

II.3 Assisting Teacher in Identifying Ideas. Does the lesson/activity include suggestions for teachers to find out what their students think about familiar situations related to a learning goal before the mathematical ideas are introduced?

II.4 Addressing Misconceptions. Does the lesson/activity explicitly address commonly held student ideas?

Overall Ratings for Category II

Middle Level Program	II.1	II.2	II.3	II.4
SpringBoard	Low/Moderate	Low	Low	Low
CMP	Moderate/High	Moderate/High	Moderate	Moderate/High
MathThematics	Moderate	Moderate	Moderate	Moderate
Glencoe	Moderate	Moderate	Low	Moderate
High School Program	II.1	II.2	II.3	II.4
SpringBoard	Low/Moderate	Low	Low	Low
Glencoe	Low	Low	Low	Low
Math Connections	Low	Low/Moderate	Low/Moderate	Low
Core-Plus	Moderate	Moderate	Moderate	Moderate

Category III: Engaging Students in Mathematics

III.1 Providing Variety of Contexts. Does the lesson/activity provide experiences with mathematics in multiple, different contexts?

III.2 Providing Firsthand Experiences. Does the lesson/activity include activities that promote firsthand experiences with the learning goal ideas, when practical?

Overall Ratings for Category III

Middle Level Program	III.1	III.2
SpringBoard	Moderate/High	Moderate
CMP	Moderate/High	Moderate/High
MathThematics	Moderate/High	Moderate/High
Glencoe	Moderate	Low/Moderate
High School Program	**III.1**	**III.2**
SpringBoard	Moderate/High	Moderate
Glencoe	Low/Moderate	Low/Moderate
Math Connections	Moderate	Moderate
Core-Plus	Moderate/High	Moderate/High

Category IV: Developing Mathematical Ideas

IV.1 Justifying Importance of Learning goal Ideas. Does the lesson/activity suggest ways to help students develop a sense of the importance and validity of mathematical concepts or procedures?

IV.2 Introducing Terms and Procedures. Does the lesson/activity introduce terms and procedures only in conjunction with experience with them and only as needed to facilitate thinking and promote effective communication?

IV.3 Representing Ideas Accurately. Does the lesson/activity include accurate and comprehensible representations of mathematical concepts, procedures, and relationships?

IV.4 Connecting Learning goal Ideas. Does the lesson/activity explicitly draw attention to appropriate connections among learning goal ideas?

IV.5 Demonstrating/Modeling Procedures. Does the lesson/activity demonstrate/model (or include suggestions for teachers on how to demonstrate/model) skills or the use of knowledge?

IV.6 Providing Practice. Does the lesson/activity provide tasks or questions for students to practice skills or use knowledge in a variety of situations?

Overall Ratings for Category IV

Middle Level Program	IV.1	IV.2	IV.3	IV.4	IV.5	IV.6
SpringBoard	Low/Moderate	Moderate/ High	Moderate	Low/ Moderate	Low/ Moderate	Moderate
CMP	Moderate	High	Moderate	Moderate	Moderate	High
MathThematics	Moderate	Moderate	Moderate	Low	Moderate/ High	Moderate/ High
Glencoe	Moderate	Low	Moderate	Low	Moderate	Moderate/ High
High School Program	**IV.1**	**IV.2**	**IV.3**	**IV.4**	**IV.5**	**IV.6**
SpringBoard	Low/ Moderate	Moderate	Moderate	Moderate	Moderate	Low/Moderate
Glencoe	Low	Low	Moderate	Low	Moderate	Moderate/ High
Math Connections	Moderate	Moderate	Moderate	Moderate	Moderate	Moderate
Core-Plus	Moderate	Moderate	Moderate	Moderate	Moderate	Moderate

Category V: Promoting Student Thinking about Mathematics

V.1 Encouraging Students to Explain Their Reasoning. Does the lesson/activity routinely include suggestions for having each student express, clarify, justify, and represent his/her ideas and how to get feedback from peers and the teacher?

V.2 Guiding Interpretation and Reasoning. Does the lesson/activity include tasks and/or question sequences that guide student interpretation and reasoning about learning goal concepts, skills, and relationships?

V.3 Encouraging Students to Think about What They've Learned. Does the lesson/activity suggest ways to have students check their own progress?

Overall Ratings for Category V

Middle Level Program	V.1	V.2	V.3
SpringBoard	Moderate/High	Moderate/High	Low
CMP	Moderate/High	High	Low/Moderate
MathThematics	Moderate	Moderate	Low/Moderate
Glencoe	Low	Low	Low
High School Program	**V.1**	**V.2**	**V.3**
SpringBoard	Moderate/High	Moderate/High	Low/Moderate
Glencoe	Low	Low	Low
Math Connections	Moderate/High	Moderate/High	Low
Core-Plus	Moderate/High	Moderate/High	Moderate

Category VI: Assessing Student Progress in Mathematics

VI.1 Aligning Assessment. Are assessment items or tasks included that match the ideas, concepts, or skills of the learning goal?

VI.2 Formal Assessment/Assessment through Applications. Does the lesson/activity include assessment tasks that require application of learning goal ideas, concepts, or skills and avoid allowing students a trivial way out, like using a formula or repeating a memorized term or rule without understanding?

VI.3 Using Informal/Embedded Assessment. Are some assessments embedded in the curriculum along the way, with advice to teachers as to how they might use the results to choose or modify activities?

Overall Ratings for Category VI

Middle Level Program	VI.1	VI.2	VI.3
SpringBoard	Moderate	Moderate/High	Low/Moderate
CMP	Moderate/High	Moderate	Low/Moderate
MathThematics	Moderate	Moderate	Low/Moderate
Glencoe	Low	Low	Low
High School Program	**VI.1**	**VI.2**	**VI.3**
SpringBoard	Moderate	Moderate/High	Low/Moderate
Glencoe	Low	Low	Low
Math Connections	Moderate	Moderate	Moderate
Core-Plus	Moderate/High	Moderate	Moderate

Category VII. Enhancing the Mathematics Learning Environment

VII.1 Providing Teacher Content Support. Does the lesson/activity help teachers improve their understanding of mathematics and its applications?

Middle Level Program	VII.1
SpringBoard	Low
CMP	Moderate
MathThematics	Low
Glencoe	Low
High School Program	VII.1
SpringBoard	Low
Glencoe	Low
Math Connections	Low/Moderate
Core-Plus	Moderate

Program Overviews of Overall Ratings:

SpringBoard Middle Level

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
L/M	M	M	L/M	L	L	L	M/H	M	L/M	L/M
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	L/M	L/M	M	M/H	M/H	L	M	M/H	L/M	L

High Ratings of Written Material: III.1 Providing a variety of contexts; V.1 Encouraging students to explain their reasoning; V.2 Guiding Interpretation and Reasoning; VI.2 Providing formal assessment opportunities through applications

Low Ratings of Written Material: II.2 Alerting teachers to student ideas; II.3 Assisting teachers in identifying ideas; II.4 Addressing Misconceptions ; IV.1 Justifying Importance of learning goal Ideas; V.3 Encouraging Students to think about what they've learned; VII.4 Providing Teacher Content Support

CMP

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
M/H	M/H	M/H	M/H	M/H	M	M/H	M/H	M/H	M	H
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	M	M	H	M/H	H	L/M	M/H	M	L/M	M

High Ratings of Written Material: I.1 Conveying Unit Purpose; I.2 Conveying Lesson Purpose ; I.3 Justifying Sequence of Activities; II.1 Specifying Prerequisite Knowledge; II.2 Alerting Teacher to Student Ideas; II.4 Addressing Misconceptions; III.1 Providing Variety of Contexts; III.2 Providing Firsthand Experiences; IV.2 Introducing Terms and Procedures IV.6 Providing Practice; V.1 Encouraging Students to Explain Their Reasoning; V.2 Guiding Interpretation and Reasoning; VI.1 Aligning Assessment; VI.3 Using Informal/Embedded Assessment

Low Ratings of Written Material: V.3 Encouraging Students to Think about What They've Learned; VI.3 Using Informal/Embedded Assessment; Category VII. Enhancing the Mathematics Learning Environment

MathThematics

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
M	M	L/M	M	M	M	M	M/H	M/H	M	M
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	L	M/H	M/H	M	M	L/M	M	M	L/M	L

High Ratings of Written Material: III.1 Providing Variety of Contexts; III.2 Providing Firsthand Experiences; IV.5 Demonstrating/Modeling Procedures; IV.6 Providing Practice

Low Ratings of Written Material: I.3 Justifying Sequence of Activities; IV.1 Justifying Importance of Learning goal Ideas; V.3 Encouraging Students to Think about What They've Learned; VI.3 Using Informal/Embedded Assessment; Category; VII.1 Providing Teacher Content Support

Glencoe Middle Level

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
M	M	M	M	M	L	M	M	L/M	M	L
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	L	M	M/H	L	L	L	L	L	L	L

High Ratings of Written Material: IV.6 Providing Practice

Low Ratings of Written Material: II.3 Assisting Teacher in Identifying Ideas; III.1 Providing Variety of Contexts; III.2 Providing Firsthand Experiences; IV.2 Introducing Terms and Procedures; V.1 Encouraging Students to Explain Their Reasoning; V.2 Guiding Interpretation and Reasoning; V.3 Encouraging Students to Think about What They've Learned; VI.1 Aligning Assessment; VI.2 Formal Assessment/Assessment through Applications; VI.3 Using Informal/Embedded Assessment; Category; VII.1 Providing Teacher Content Support

SpringBoard High School

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
L/M	M	M	L/M	L	L	L	M/H	M	L/M	M
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	M	M	L/M	M/H	M/H	L/M	M	M/H	L/M	L

High Ratings of Written Material: III.1 Providing a variety of contexts; V.1 Encouraging students to explain their reasoning; V.2 Guiding Interpretation and Reasoning; VI.2 Providing formal assessment opportunities through applications

Low Ratings of Written Material: I.1 Conveying Unit Purpose; II.2 Alerting teachers to student ideas; II.3 Assisting teachers in identifying ideas; II.4 Addressing Misconceptions; IV.1 Justifying Importance of learning goal Ideas; V.3 Encouraging Students to think about what they've learned; VII.4 Providing Teacher Content Support

Glencoe High School

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
M	L	L/M	L	L	L	L	L/M	L/M	L	L
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	L	M	M/H	L	L	L	L	L	L	L

High rating of Written Material: IV.6 Providing Practice

Low Rating of Written Material: I.2 Conveying Lesson Purpose ; II.1 Specifying Prerequisite Knowledge; II.2 Alerting Teacher to Student Ideas; II.3 Assisting Teacher in Identifying Ideas;II.4 Addressing Misconceptions; IV.1 Justifying Importance of Learning goal Ideas; IV.2 Introducing Terms and Procedures; IV.4 Connecting Learning goal Ideas; V.1 Encouraging Students to Explain Their Reasoning; V.2 Guiding Interpretation and Reasoning; V.3 Encouraging Students to Think about What They've Learned; VI.1 Aligning Assessment; VI.2 Formal Assessment/Assessment through Applications; VI.3 Using Informal/Embedded Assessment; Category VII. Enhancing the Mathematics Learning Environment; VII.1 Providing Teacher Content Support

Math Connections

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
M	M	M	L	L/M	L/M	L	M	M	M	M
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	M	M	M	M/H	M/H	L	M	M	M	L/M

High Ratings of Written Material: V.1 Encouraging Students to Explain Their Reasoning; V.2 Guiding Interpretation and Reasoning

Low Ratings of Written Material: II.1 Specifying Prerequisite Knowledge; II.2 Alerting Teacher to Student Ideas; II.3 Assisting Teacher in Identifying Ideas;II.4 Addressing Misconceptions; VII.1 Providing Teacher Content Support

Core-Plus

I.1	I.2	I.3	II.1	II.2	II.3	II.4	III.1	III.2	IV.1	IV.2
M	M/H	M	M	M	M	M	M/H	M/H	M	M
IV.3	IV.4	IV.5	IV.6	V.1	V.2	V.3	VI.1	VI.2	VI.3	VII.1
M	M	M	M	M/H	M/H	M	M/H	M	M	M

High Ratings of Written Material: I.2 Conveying Lesson Purpose; III.1 Providing Variety of Contexts; III.2 Providing Firsthand Experiences; V.1 Encouraging Students to Explain Their Reasoning; V.2 Guiding Interpretation and Reasoning; VI.1 Aligning Assessment

Low Ratings of Written Material: N/A

VII: Analysis of Curriculum Program Alignment to College Board Standards for College Success

Description of Analysis Procedure

The College Board Standards for College Success provide a comprehensive and coherent plan for mathematics, reading, and writing instruction for grades 6-12 designed to prepare students for success in college. They are focused on building analytical, problem-solving, and critical thinking skills. Based on the "Knowledge and Skills for University Success", a three-year project developed by the Center for Educational Policy Research at the University of Oregon for the Association of American Universities in partnership with the Pew Charitable Trusts, the College Board Standards for College Success are unique in providing a roadmap of the critical thinking skills students need to develop as they progress from 6th grade through 12th grade in order to be prepared to succeed in college.The College Board Standards for College Success were developed in consultation with national content experts and educators from college, high school and middle school. The development committees drew on nationally recognized standards documents and the standards implicit in the Advanced Placement Program and SAT programs as well as college placement exams. The College Board Standards for College Success provide a blueprint for high expectations for student progress from middle through high school, preparing more students for the rigor of Advanced Placement Program and college courses.The College Board Standards for College Success are comprised of the following:

- Standards: the critical thinking skills expected as students enter college or AP coursework.
- Strands: coherent themes across Levels that organize the content
- Levels: a progressive framework in skills and knowledge from middle school through high school. Each Level generally corresponds to an academic year's worth of coursework. Levels provide a roadmap that prepares more students to advance to college level work upon high school graduation or enrollment in AP courses.
- Performance Expectations: a series of expectations for what students should know and be able to do as they progress through each level from middle school through high school. (Information on College Board Standards for College Success retreived from http://www.collegeboard.com/springboard/standards.html)

This analysis focuses on the standards of Algebra and Data Analysis & Probability as the concepts within these two standards are the foundational ideas needed for success in the AP Calculus and AP Statistic courses.

Within these standards, the following strands were included in the alignment process:
Standard: Alegbra

Strands

- Patterns and Relations
- Variables, Exressions, and Inequalities
- Functions with Growth and Change
- Problem Solving
- Representations

Standard: Data Analysis and Probability

Strands

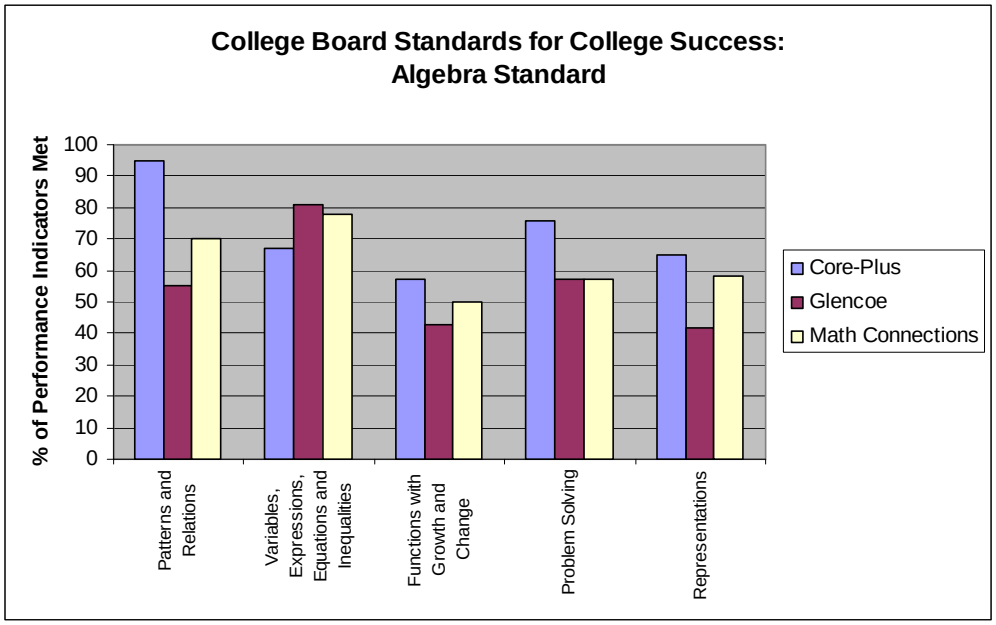
- Data Collection
- Data Exploration
- Data Interpretation
- Proobability Concepts and Applications
- Problem Solving
- Representations

Within each of the strands, three to six levels represent a progressive framework of skills and knowledge. Two to five performance indicators further clarify what students should know and be able to do at that particular level. The primary instructional goals of each lesson in the three middle level and three high school programs were analyzed and aligned to the performance indicator level of the College Board Standards for College Success.

The six levels represent a typical progression of skills and knowledge acquired within the 6-12 curriculum. Some overlap of the levels is expected as the programs analyzed represent a three year middle level program. The three high school programs analyzed represent a three year core-curriculum which does not include a formal pre-calculus course.

Results of the Alignment Analysis:

Algebra Strand/High School



This graph depicts the percent of Level I-VI performance indicators addressed for each Strand of the Algebra Standard. With the exception of the Variables, Expressions, Equations and Inequality Strand, the Core-Plus program addressed a higher percentage of performance indicators than Glencoe and Math Connections. Additional detail on percentage of lessons aligned to each of the performance indicators within a strand follows below.

The Levels and Performance Expectations of each content strand describe a progressive development of skills. The process Strands (Problem Solving and Representations for this alignment process), involve the same processes with increasingly complex content. For this reason, a careful review of the Performance Expectations addressed within a program may reveal gaps in the progression of ideas.

The College Board Standards for College Success also inlcude these Performance Expectations grouped by the course with Middle Level I and II representing a "pre-algebra" middle level program and four additional courses of Algebra, Geometry, Algebra II and Advanced Mathematics.

In comparing the Level I – VI Performance Expectations in the Algebra Standards to the Course Performance Expectations, Level I is considered pre-algebra and Level II and III, Algebra. The majority of the three high school programs align at grade level for the Algebra 1 strands. Level IV and V is content considered to be at the Algebra II course level and is covered to a lesser extent within the programs with many of the algebra 2 lessons still addressing algebra 1 strands.

Standard: Algebra ***Strand: Patterns and Relations***

Core-Plus

- 35% of Lessons addressed a Performance Expectation in this Strand
- 95% of the Patterns and Relations Performance Expectations addressed

Glencoe

- 4.9% of Lessons addressed a Performance Expectation in this Strand
- 55% of the Patterns and Relations Performance Expectations addressed

Math Connections

- 16.6 % of Lessons addressed a Performance Expectation in this Strand
- 70% of the Patterns and Relations Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Patterns and Relations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Recognize and describe linear relationships and patterns, and distinguish between linear and nonlinear patterns. C-Plus 3.2 % Glencoe .4% M Conn .64%	Recognize and describe multistep linear relationships and patterns. C-Plus 2.1% M Conn .64%	Recognize and describe absolute value, piecewise-defined, and quadratic patterns and relationships. C-Plus 1.6% Glencoe .4%	Recognize and describe rational, radical, and polynomial patterns and relationships. C-Plus 1% M Conn .64	Recognize and describe exponential, logarithmic, and recursive patterns and relationships. CPlus 3.2% M Conn 2.5%	Recognize, create, and describe trigonometric, polar, and parametric patterns and relationships. C-Plus 1.6% Glencoe .7% M Conn 2.5%
2)	Create and extend simple linear patterns. C-Plus .5% Glencoe .4%	Recognize a rate as a comparison of the change in one quantity relative to a one-unit change in a second quantity. C-Plus 4.8% Glencoe .7% M Conn 1.9%	Create and extend absolute value, piecewise-defined, and quadratic patterns and relationships. C-Plus 1 % Glencoe .7% M Conn .64%	Recognize the relationship between a function and its inverse, verbally, numerically, graphically, and symbolically. Glencoe .4% M Conn .64%	Create and extend exponential, logarithmic, and recursive patterns and relationships, and their inverses. C-Plus 2.7% Glencoe .4% M Conn 1.9%	Create and extend trigonometric, polar, and parametric patterns and relationships, and their inverses. C-Plus 1.6% M Conn 1.3%
3)	Generalize simple linear patterns with a verbal or symbolic rule. C-Plus 1.6% Glencoe .4% M Conn .64%	Create and extend multistep linear patterns. C-Plus .5% Glencoe .4%	Generalize absolute value, piecewise-defined, and quadratic patterns and relationships with a verbal or symbolic rule. C-Plus .5 % M Conn. 64%	Create and extend rational, radical, and polynomial patterns and relationships, and their inverses. C-Plus 1%	Generalize exponential, logarithmic, and recursive patterns and relationships with a verbal or symbolic rule. C-Plus 2.1% M Conn 1.3%	Generalize trigonometric, polar, and parametric patterns and relationships with a verbal or symbolic rule. C-Plus .5%
4)	NA	Generalize multistep linear patterns with a verbal or symbolic rule. C-Plus 3.7% Glencoe .7% M Conn .64%	NA	Generalize rational, radical, and polynomial patterns and relationships with a verbal or symbolic rule. C-Plus 1%	NA	NA

Standard: Algebra Strand: Variables, Expressions, Equations and Inequalities

Core-Plus

- 49.2% of Total Lessons addressed a Performance Expectation in this Strand
- 67% of the Variables, Expressions, Equations and Inequalities Performance Expectations addressed

Glencoe

- 61.8% of Total Lessons addressed a Performance Expectation in this Strand
- 81% of the Variables, Expressions, Equations and Inequalities Performance Expectations addressed

Math Connections

- 43.3 % of Total Lessons addressed a Performance Expectation in this Strand
- 78% of the Variables, Expressions, Equations and Inequalities Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Variables, Expressions, Equations and Inequalities Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Identify the constants and variable terms in simple linear relationships and patterns, and identify the independent and dependent variables in linear relationships. C-Plus 1 % Glencoe .7% M Conn 2.5%	Create linear expressions, equations, and inequalities that represent algebraic situations, including systems of equations and inequalities. C-Plus 5.9% Glencoe 5.3% M Conn 5.7%	Create absolute value, piecewise-defined, and quadratic expressions and equations that represent algebraic situations. C-Plus 1% Glencoe 1.8% M Conn .6%	Create rational, radical, and polynomial expressions and equations that represent algebraic situations. C-Plus 2.7% Glencoe 1.8%	Create exponential, logarithmic, and recursive expressions and equations that represent algebraic situations. C-Plus 4.3% Glencoe .7% M Conn 1.3%	Create trigonometric, polar, and parametric expressions and equations that represent algebraic situations. C-Plus 2.1% M Conn 1.3%
2)	Set up and solve proportions involving whole numbers. C-Plus 1.6% Glencoe 2.5% M Conn 3.8%	Evaluate and simplify linear expressions, linear equations, linear inequalities, and systems of linear equations and inequalities. C-Plus 3.7% Glencoe 2.8% M Conn 3.2%	Evaluate absolute value, piecewise-defined, and quadratic expressions. C-Plus 1% Glencoe .7% M Conn 1.3%	Simplify and evaluate rational, radical, and polynomial expressions and equations. Glencoe 9.3%	Graph and interpret the graphs of exponential, logarithmic, and recursive functions. C-Plus 2.7% Glencoe .4% M Conn 2.5%	Simplify and evaluate trigonometric, polar, and parametric expressions and equations. C-Plus 1% Glencoe 1.8% M Conn 1.9%
3)	Create linear expressions and equations that represent algebraic situations. C-Plus 2.7% Glencoe 1.1% M Conn 1.3%	Solve linear equations, linear inequalities, and systems of linear equations and inequalities. C-Plus 5.9% Glencoe 8.8% M Conn 5.7%	Simplify absolute value, piecewise-defined, and quadratic expressions and equations. Glencoe 1.4%	Solve rational, radical, and polynomial equations and systems of these equations. C-Plus 2.1% Glencoe 2.5% M Conn .6%	Simplify and evaluate exponential, logarithmic, and recursive expressions and equations. C-Plus .5% Glencoe 1.8% M Conn 2.5%	Solve trigonometric, polar, and parametric equations and systems of these equations. C-Plus 1% Glencoe 1.4% M Conn 2.5%

Continued:

Standard: Algebra Strand: Variables, Expressions, Equations and Inequalities

Summary of % of Lessons Aligned to Individual Variables, Expressions, Equations and Inequalities Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
4)	Evaluate and simplify linear expressions. M Conn .6%	Identify the constants and variable terms in linear equations, linear inequalities, and systems of linear equations and inequalities, including indep. and depen. variables. M Conn 1.9%	Solve absolute value, piecewise- defined, and quadratic equations and systems of these equations. C-Plus 2.7% Glencoe 6.7% M Conn 1.3%	Identify the constants and variable terms in rational, radical, and polynomial functions. Glencoe .7%	Solve exponential, logarithmic, and recursive equations and systems of these equations. C-Plus 1.6% Glencoe 1.8% M Conn .6%	NA
5)	Solve one-step linear equations. Glencoe 1.8% M Conn .6%	Apply the commutative, associative, and distributive properties to simplify and solve algebraic expressions and equations. Glencoe 6% M Conn .6%	Identify the constants and variable terms in absolute value, piecewise- defined, and quadratic equations and inequalities, including the indepen. and depen. variables. C-Plus 1%	Use properties of inverses to solve algebraic problems. Glencoe .4%	NA	NA

Standard: Algebra ***Strand: Functions with Growth and Change***

Core-Plus

- 12% of Total Lessons addressed a Performance Expectation in this Strand
- 57% of the Functions with Growth and Change Performance Expectations addressed

Glencoe

- 4.2% of Total Lessons addressed a Performance Expectation in this Strand
- 43% of the Functions with Growth and Change Performance Expectations addressed

Math Connections

- 9.6% of Total Lessons addressed a Performance Expectation in this Strand
- 50% of the Functions with Growth and Change Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Functions with Growth and Change Inequalities Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	NA	NA	NA	Solve problems involving rational, radical, and polynomial functions. C-Plus 2.1%	Identify the constants and variable terms in exponential, logarithmic, and recursive functions.	Identify the constants and variable terms in trigonometric, polar, and parametric functions.
2)	NA	NA	NA	Determine whether a given relation is a function or not based on its definition or on its graphical representation. C-Plus .5% Glencoe .4% M Conn 1.3%	Understand the relationship between exponential and logarithmic functions. M Conn .6%	Perform the composition of linear and trigonometric functions.
3)	NA	NA	NA	Graph and interpret the graphs of absolute value, piecewise-defined, and quadratic functions, in terms of zeros, intercepts, and overall qualitative behavior. C-Plus 1.6% M Conn 1.3% Glencoe 2.1%	Demonstrate an understanding of recursively defined functions. C-Plus 2.7% Glencoe .4% M Conn .6%	Demonstrate an understanding of trigonometric, inverse trigonometric, polar, and parametric functions. C-Plus 1.6% Glencoe .7% M Conn 1.9%
4)	NA	NA	NA	Relate the changes in functional values to changes in the domain values. C-Plus .5% Glencoe .4%	Understand exponential, logarithmic, and recursive functions, and sequences and series. C-Plus 2.1% M Conn 3.2%	Find the sum of terms in an arithmetic series and in finite and infinite geometric series. C-Plus .5%
5)	NA	NA	NA	Demonstrate an understanding of the composition of linear and quadratic functions. Glencoe .4% M Conn .6%	Demonstrate an understanding of the composition of linear, exponential, and logarithmic functions.	NA

Standard: Algebra ***Strand: Problem Solving***

Core-Plus

- 42% of Total Lessons addressed a Performance Expectation in this Strand
- 76% of the Problem Solving Performance Expectations addressed

Glencoe

- 17% of Total Lessons addressed a Performance Expectation in this Strand
- 57% of the Problem Solving Performance Expectations addressed

Math Connections

- 17.8% of Total Lessons addressed a Performance Expectation in this Strand
- 57% of the Problem Solving Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Problem Solving Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Describe problem situations involving simple linear relationships and patterns, and identify appropriate problem-solving strategies for such situations. C-Plus 2.7%	Describe problem situations involving linear equations, linear inequalities, and systems of linear equations and inequalities, and identify appropriate problem-solving strategies for such situations. C-Plus 3.2 % M Conn .6%	Describe problem situations involving absolute value, piecewise-defined, and quadratic equations, and identify appropriate problem-solving strategies for such situations. C-Plus 1.6%	Describe problem situations involving rational, radical, and polynomial functions, and identify appropriate problem-solving strategies for such situations. C-Plus .5%	Describe problem situations involving exponential, logarithmic, and recursive functions, and identify appropriate problem-solving strategies for such situations. C-Plus 1%	Describe problem situations involving trigonometric, inverse trigonometric, polar, and parametric functions, and identify appropriate problem-solving strategies for such situations. C-Plus .5% Glencoe .4% M Conn 1.9%
2)	Solve problems involving simple linear relationships and patterns. C-Plus 2.7% M Conn .6%	Solve problems involving rates, showing the relationships in algebraic and geometric forms. C-Plus 2.7% Glencoe .4%	Solve problems involving absolute value, piecewise-defined, and quadratic equations. C-Plus 3.2% Glencoe 1.1% M Conn 1.9%	Apply the properties of matrices to solving systems of equations. C-Plus 2.7% Glencoe 1.4% M Conn .6%	Solve problems involving exponential, logarithmic, and recursive functions and sequences and series. C-Plus 4.3% Glencoe 1.1% M Conn 3.2%	Solve problems involving the composition of linear and trigonometric functions.

Continued

Standard: Algebra ***Strand: Problem Solving***

Summary of % of Lessons Aligned to Individual Problem Solving Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
3)	Apply order of operations to algebraic expressions in problem solving. C-Plus .5% Glencoe 1.1%	Solve problems involving linear equations, linear inequalities, and systems of linear equations and inequalities. C-Plus 5.3% Glencoe 4.6% M Conn 6.4%	NA	Solve problems involving the composition of linear and quadratic functions. M Conn .6%	Solve problems involving the composition of linear, exponential, and logarithmic functions. Glencoe .4%	Solve problems involving trigonometric, inverse trigonometric, polar, and parametric functions. C-Plus 2.1% Glencoe 1.4% M Conn 1.3%
4)	NA	Solve multistep problems involving linear equations and inequalities. C-Plus 3.2 % Glencoe 2.5% M Conn 1.3%	NA	Solve problems involving linear programming and matrices. C-Plus 3.2 % Glencoe 1.4% M Conn 3.2%	Apply the properties of logarithms and exponents and the equivalence between $y=a^x$ and $x = \log_a y$ to the solution of problems involving logarithmic and exponential equations. Glencoe 1.4% M Conn 1.3%	Solve problems involving the sum of terms in an arithmetic series and in finite and infinite geometric series.

Standard: Algebra ***Strand: Representations***

Core-Plus

- 38.5% of Total Lessons addressed a Performance Expectation in this Strand
- 65% of the Representations Performance Expectations addressed

Glencoe

- 23.7% of Total Lessons addressed a Performance Expectation in this Strand
- 42% of the Representations Performance Expectations addressed

Math Connections

- 46% of Total Lessons addressed a Performance Expectation in this Strand
- 58% of the Representations Performance Expectations addressed

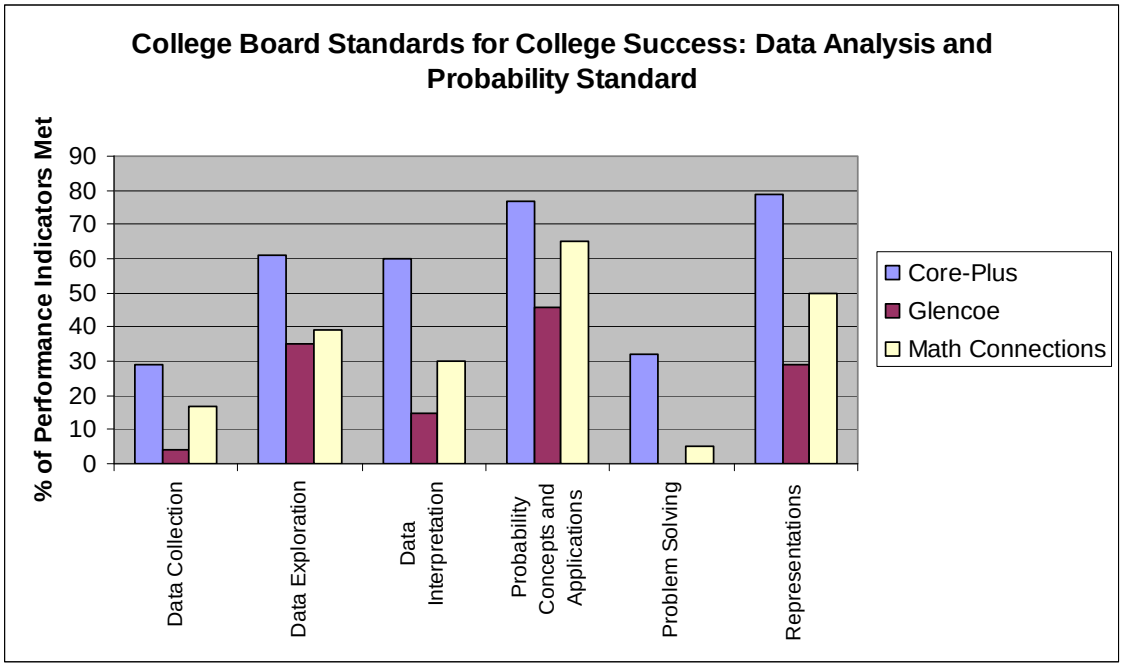
Summary of % of Lessons Aligned to Individual Representations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Recognize or create appropriate representations of simple linear relationships and patterns, including verbal, tabular, graphical, and symbolic representations. C-Plus 3.7% Glencoe 1.8% M Conn 1.3%	Recognize and create appropriate representations of linear equations, linear inequalities, and systems of linear equations and inequalities, including verbal, tabular, graphical, and symbolic representations. C-Plus 5.9% Glencoe 6.4 % M Conn 8.3%	Recognize and create appropriate representations of absolute value, piecewise-defined, and quadratic functions. C-Plus 2.1% Glencoe 6.4% M Conn 1.3	Recognize and create appropriate representations, including matrices, of rational, radical, and polynomial functions, including verbal, tabular, graphical, and symbolic representations. C-Plus 2.1% Glencoe 3.2 % M Conn .6%	Recognize and create appropriate representations, including matrices, of exponential, logarithmic, and recursive functions, including verbal, tabular, graphical, and symbolic representations. C-Plus 3.7% Glencoe 1.4% M Conn 4.5%	Recognize and create appropriate representations of trigonometric inverse trigonometric, polar, and parametric functions, including verbal, tabular, graphical, and symbolic representations. C-Plus 2.1% Glencoe 1.1% M Conn 5.1%
2)	Describe the characteristics and defend the appropriateness of alternative representations of simple linear relationships and patterns.	Describe the characteristics and defend the appropriateness of alternative representations of linear equations, linear inequalities and systems of equations and inequalities. C-Plus 1% M Conn .6%	Describe the characteristics and defend the appropriateness of alternative representations of absolute value, piecewise-defined and quadratic functions. C-Plus .5% Glencoe .4% M Conn .6%	Describe the characteristics and defend the appropriateness of alternative representations of rational, radical, and polynomial functions. Glencoe .4% M Conn .6%	Describe the characteristics and defend the appropriateness of alternative representations of exponential, logarithmic, and recursive functions, and sequences and series. M Conn .6%	Describe the characteristics and defend the appropriateness of alternative representations of trigonometric, inverse trigonometric, polar, and parametric functions. M Conn .6%
3)	Identify and recognize equivalent verbal, tabular, graphical, and symbolic representations of simple linear relationships and patterns. C-Plus 2.1 %	Identify and recognize equivalent representations, including verbal, tabular, graphical, and symbolic representations, of linear equations and inequalities and systems of linear equations. C-Plus 3.2 % Glencoe 1.1% M Conn .6%	Recognize, apply, and translate among equivalent representations of absolute value, piecewise-defined, and quadratic functions, including verbal, graphical, and symbolic representations. C-Plus 1.6% Glencoe 1.4%	Identify, recognize, and apply equivalent representations of rational, radical, and polynomial functions, including verbal, tabular, graphical, and symbolic representations.	Identify and apply equivalent representations of exponential, logarithmic, and recursive functions, including verbal, tabular, graphical, and symbolic representations.	Identify and apply equivalent representations of trigonometric, inverse trigonometric, polar, and parametric functions, including verbal, tabular, graphical, and symbolic representations. C-Plus .5% M Conn 1.9%

Continued:

Standard: Algebra ***Strand: Representations***

Summary of % of Lessons Aligned to Individual Representations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
4)	Use and translate among equivalent representations of simple linear relationships and patterns. C-Plus 1%	Use and translate among equivalent representations of linear equations, linear inequalities, and systems of linear equations and inequalities. C-Plus 4.8% Glencoe .4% M Conn 1.9%	Compare and contrast equivalent representations of absolute value, piecewise-defined, and quadratic functions. C-Plus .5%	Compare and contrast equivalent representations of rational, radical, and polynomial functions.	Compare and contrast equivalent representations of exponential, logarithmic, and recursive functions, and sequences and series. C-Plus .5%	Compare and contrast equivalent representations of trigonometric, inverse trigonometric, polar, and parametric functions.
5)	Compare and contrast equivalent representations of simple linear relationships and patterns. C-Plus .5%	Compare and contrast equivalent representations of linear equations, linear inequalities, and systems of linear equations and inequalities. M Conn .6%	NA	NA	NA	NA

Data Analysis & Probability Strand/High School



This graph depicts the percent of Level I-VI Performance Expectations addressed for each Strand of the Data Analysis and Probability Standard. For all strands, the Core-Plus program addressed a higher percentage of Performance Expectations than Glencoe and Math Connections. Additional detail on percentage of lessons aligned to each of the Performance Expectations within a strand follows below.

In comparing the Level I – VI Data Analysis and Probability Performance Expectations to the Course Performance Expectations, the majority of the Level I, II and III expectations are considered middle level content. The main focus of all three programs is at Level I - III, so although Data Analysis concepts were covered, they were done at a lower grade level than expected.

Standard: Data Analysis and Probability ***Strand: Data Collection***

Core-Plus

- 11.2% of Total Lessons addressed a Performance Expectation in this Strand
- 29% of the Data Collection Performance Expectations addressed

Glencoe

- .4% of Total Lessons addressed a Performance Expectation in this Strand
- 4% of the Data Collection Performance Expectations addressed

Math Connections

- 4.5% of Total Lessons addressed a Performance Expectation in this Strand
- 17% of the Data Collection Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Data Collection Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Describe data collection processes for gathering univariate categorical data on a small population. C-Plus .5% M Conn .6%	Describe data collection processes for gathering univariate numerical data on a small population. M Conn .6%	Describe data collection processes for gathering bivariate data (two variables for each case).	Describe both a census and a simple random sample from a population. Recognize and describe the differences between population parameters and sample statistics. C-Plus .5%	Recognize and describe the differences between observational and experimental studies.	Describe stratified random sampling in sample survey design and possible other types of more complex probability sampling, such as cluster or multistage sampling. Recognize situations in which more complex types of probability sampling are needed.
2)	Distinguish between "Cases," the units that are measured (e.g., an individual student), and "Variables," the characteristic that is measured (e.g., favorite food).	Describe a method for measuring the variable of interest when gathering univariate numerical data on a small population.	Recognize three types of bivariate data: the variables may both be categorical, both numerical, or one numerical and one categorical.	Describe techniques for drawing a simple random sample (e.g., giving every unit in the population an ID number, writing the numbers on identical slips of paper, and drawing slips of paper at random from a bowl to select the units of the sample). C-Plus .5% Glencoe .4%	Recognize that experiments use randomization to balance the variation of some factors that may affect experimental results.	Describe blocking in experimental design, and explain how it reduces variation. Recognize situations in which blocked experimental designs are needed.
3)	Collect univariate categorical data on a small population by classifying observations into one of several categories (e.g., favorite foods of the students in a given classroom). C-Plus .5%	Collect univariate numerical data on a small population (e.g., statistical investigations of characteristics, such as height or arm span of students in a given classroom or the number of books owned by boys and girls in the class). M Conn .6%	Recognize two settings for bivariate analysis: two measurements on each subject in a single small population, or one measurement on each subject in two or more small populations (e.g., statistical investigations may be made of such situations as arm span versus height for students in a single classroom or heights of students in two or more classrooms).	Describe types of nonrandom sampling, such as convenience (haphazard) sampling and judgment sampling.	Design a simple comparative experiment to answer a research question; determine treatments and methods of measuring outcome variables; include randomization of units to treatments, carry out the experiment, and collect the data.	Design a survey for a stratified random sample from a population, and collect the data (e.g., stratify the school into grade levels and draw random samples from each grade level). Organize the collected survey data so that it can be effectively displayed in tables or graphs.

Continued:

Standard: Data Analysis and Probability ***Strand: Data Collection***

Summary of % of Lessons Aligned to Individual Data Collection Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
4)	NA	Organize collected data so that it can be effectively displayed. C-Plus 1.6% M Conn 2.5%	Collect all three types of bivariate data. C-Plus 3.7%	Draw a small simple random sample from a population, collect numerical and/or categorical data on sampled subjects, and organize the collected sample data so that it can be effectively displayed in tables or graphs.	Organize the data from an experiment so that it can be effectively displayed in tables or graphs.	Design and carry out a blocked experiment to compare two or more treatments (e.g., compare sitting and standing pulse rates of students while controlling for gender). Effectively summarize and display the data.
5)	NA	NA	Organize the collected data so that it can be effectively displayed in tables or graphs. C-Plus 3.7%	NA	NA	NA

Standard: Data Analysis and Probability ***Strand: Data Exploration***

Core-Plus

- 34.2% of Total Lessons addressed a Performance Expectation in this Strand
- 61% of the Data Exploration Performance Expectations addressed

Glencoe

- 4.2% of Total Lessons addressed a Performance Expectation in this Strand
- 35% of the Data Exploration Performance Expectations addressed

Math Connections

- 21% of Total Lessons addressed a Performance Expectation in this Strand
- 39% of the Data Exploration Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Data Exploration Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Summarize the collected data, beginning with counts and then moving to proportions and percentages. C-Plus 2.7%	Identify and describe the processes and techniques for describing the distribution of values in a univariate numerical data set, including the use of graphical displays such as dot plots, stem plots, and histograms. C-Plus 4.8% Glencoe .4% M Conn 3.2%	Identify, describe, and construct appropriate displays for studying the association between two variables: two-way tables for categorical versus categorical; parallel box plots (or other appropriate comparative graphs) for numerical versus categorical; and scatterplots for numerical versus numerical. C-Plus 3.7% Glencoe 1.1% M Conn 1.3%	Recognize that summary numbers, such as center and spread computed from a random sample, are likely to differ from the same summary numbers computed from a census of the same population.	Plot experimental data, and look for patterns in the plots. For example, check for an association between two numerical variables by making a scatterplot and examining it for patterns. C-Plus 2.7% Glencoe .4% M Conn 1.9%	Use the difference between proportions to describe and measure the association between two categorical variables.
2)	Recognize that the categories might be ordered and that the category labeled "small size" (e.g., T-shirt size) may have the largest frequency of use.	Describe the overall shape of the distribution, including clumps, bumps, and gaps in the data, extremes, and range. C-Plus 2.7% Glencoe .4% M Conn 3.8%	Recognize and describe the similarities and differences among the three types of displays: for example, outliers affect the degree of association exhibited in all three types; tables of categorical data and scatterplots each can show trends. C-Plus 1% M Conn .6%	Use simulation to explore the variability of characteristics of random samples, such as shape, center, and spread in samples drawn from a known population, and to study the sampling distributions of means and proportions.	Compute a correlation between two numerical variables as a measure of strength of association. C-Plus 2.1% Glencoe .4% M Conn .6%	Compute a Chi-squared statistic to check for association in a two-way table, and use stimulation to study the distribution of the Chi-squared statistic.

Continued:

Standard: Data Analysis and Probability ***Strand: Data Exploration***

Summary of % of Lessons Aligned to Individual Data Exploration Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
3)	Describe the distribution of values in a univariate categorical data set through the use of graphical displays, such as bar charts and pie charts. C-Plus 3.2% Glencoe .4%	Recognize, be bale to compute, and interpret measures of center, such as the mean, the median, and the mode, and simple measures of spread, such as range and interquartile range. C-Plus 4.3% Glencoe 1.1% M Conn 4.5%	NA	Use simulation to explore what kinds of sample results might be expected under various assumptions about the population (e.g., if 35 percent of the students in the school like a particular school policy, what would the distribution of sample proportions look like for samples of 25?). C-Plus 1%	Use a lest-squares regression line or a median-fit line to describe the type of association shown in a scatterplot. C-Plus 2.1% Glencoe .4% M Conn 2.5%	Use the difference between means or medians to describe and measure the association between numerical and categorical variables: for example, does the mean score in the second group differ from the mean score in the first group?
4)	Compare proportions/percentages in each category of a univariate categorical data set, and determine ordinal rankings such as most and least. C-Plus .5%	Relate measures of center and spread to graphical representations of data. C-Plus 2.1% M Conn 2.5%	NA	Compare results from random and non-random samples of the same population.	Use a simple randomization test to compare two means in a designed experiment in which treatments were randomly assigned to experimental units.	Use simulation to create confidence intervals for sample proportions and sample means and to study the effect of sample size on sampling distributions and on confidence intervals.
5)	NA	NA	NA	NA	NA	Use transformations of numerical variables to modify their shape, center, and spread. C-Plus .5%

Standard: Data Analysis and Probability ***Strand: Data Interpretation***

Core-Plus

- 10.2% of Total Lessons addressed a Performance Expectation in this Strand
- 60% of the Data Interpretation Performance Expectations addressed

Glencoe

- 1.8% of Total Lessons addressed a Performance Expectation in this Strand
- 15% of the Data Interpretation Performance Expectations addressed

Math Connections

- 6.4% of Total Lessons addressed a Performance Expectation in this Strand
- 30% of the Data Interpretation Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Data Interpretation Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Use the results of a categorical data collection to answer the question that motivated the gathering of data. C-Plus .5%	Recognize the difference between numerical and categorical data: for example, that a count, in the context of numerical data, represents the measurement on a single person and not a summary statistic for the class. M Conn .6%	Discuss the association between two numerical characteristics measured on the same population. C-Plus 1.6% Glencoe .7% M Conn .6%	Compare the advantages and disadvantages of taking a census, drawing a random sample, or using a nonrandom sample. C-Plus 1%	Distinguish between associations and a casual relationship between variables. C-Plus 1.6%	Recognize that sample-to-sample variability must be taken into account when making inferences from sample data to a population. C-Plus .5%
2)	Critique an analysis of univariate categorical data, such as a bar graph or a pie chart found in a newspaper of magazine, with regard to the data display used and the reasonableness of the conclusions drawn.	Use the results of a numerical data collection to answer a question of interest, and discuss what results mean in the context of the question of interest. Glencoe .4% M Conn 1.9%	Compare and contrast the distributions of some numerical characteristic measured in two or more populations. C-Plus .5%	Recognize when it is appropriate to draw conclusions about a population from sample data, and recognize the importance of sample-to-sample variability in interpreting results from sample data.	Interpret the correlations between two numerical variables. C-Plus 1.6%	Explain how confidence intervals can be used to summarize sample survey or experimental results, and interpret confidence intervals and confidence levels. C-Plus .5% M Conn .6%
3)	NA	Critique an analysis of numerical data, such as a line graph or a histogram found in an article or book, particularly the reasonableness of the conclusions drawn.	Critique an analysis of bivariate data, such as a scatterplot found in an article or book, particularly the reasonableness of the conclusions drawn.	Identify possible sources of bias in sample data: for example, recognize factors that may affect sample results, such as the way survey questions are worded or the method used to draw the sample. C-Plus .5% Glencoe .7% M Conn .6%	Interpret the slope and intercept of a regression line. C-Plus .5% M Conn 1.9%	Recognize the effects of a linear transformation on the center, shape, and spread of distribution. C-Plus 1%
4)	NA	NA	NA	Critique an analysis of sample data, such as the appropriateness of the conclusions drawn from the sample because of the way the sample was collected from the population. C-Plus .5%	Interpret the results of randomization test of means.	Recognize that data scales are chosen for convenience of analyzing and presenting information.

Standard: Data Analysis and Probability ***Strand: Probability Concepts and Applications***

Core-Plus

- 36.4% of Total Lessons addressed a Performance Expectation in this Strand
- 77% of the Probability Concepts and Applications Performance Expectations addressed

Glencoe

- 9.5% of Total Lessons addressed a Performance Expectation in this Strand
- 46% of the Probability Concepts and Applications Performance Expectations addressed

Math Connections

- 18.5% of Total Lessons addressed a Performance Expectation in this Strand
- 65% of the Probability Concepts and Applications Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Probability Concepts and Applications Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Conduct experiments when known structure (e.g., with coins, dice, spinners, or cards) to estimate the likelihood of simple outcomes. C-Plus 2.1% M Conn .6%	Collect sample or experimental data to estimate probabilities of outcomes for which the theoretical probability is unknown, (e.g., the probability that a randomly selected person from the school will have a pet dog). Summarize the data in a table or graph. C-Plus 2.7%	Describe the set of possible outcomes (sample space) of a random experiment. Define events as subsets of the set of possible outcomes (e.g., in the experiment of tossing a die, describe the event "the outcome is even" as a subset of the possible outcomes). Recognize complementary and mutually exclusive events. C-Plus 1% Glencoe 1.1% M Conn 1.3%	Distinguish between independent and dependent events, and understand the basic idea of conditional probability. C-Plus 1.6% Glencoe .7% M Conn .6%	Understand the notion of a random variable and its probability distribution, such as that of the number of heads when three coins are tossed. C-Plus 3.2% Glencoe .7% M Conn .6%	Use combinations or permutations to count outcomes in more complex sample spaces. Glencoe .7% M Conn 1.3%
2)	Relate the estimate (empirical probability) to the structure of the device (theoretical probability), such as the sizes of the colored regions on the spinner or the number of red cards in the deck. C-Plus 1%	Recognize that if the estimated probability is to be reliable, the collected data must satisfy certain conditions: for example, the data must form a representative sample of the population.	Assign probabilities, either theoretical or empirical, to events. C-Plus 2.1% Glencoe 1.8% M Conn 1.3%	Understand and use the additive law of probability, and understand and use the multiplicative law of probability. C-Plus .5% Glencoe 1.4% M Conn 1.3%	Compute expected values for experimental situations involving counts (e.g., the expected number of heads in 10 tosses of a fair coin). C-Plus 4.3% M Conn 1.3%	Develop and use the binomial probability distribution C-Plus 1% M Conn 1.3%
3)	Identify the probability of an event as a number between 0 and 1, and compare the likelihood of different outcomes (more likely, less likely); for example, compare outcomes from spinners with differently sized regions. C-Plus .5% M Conn .6%	Express a probability as a fraction, a decimal, or a percent. C-Plus 2.1% Glencoe .7% M Conn 1.9%	Use Venn diagrams to show relationships among events and to help establish probabilities of these events. M Conn 1.3%	Use tree diagrams to find probabilities of compound events. Glencoe .7% M Conn .6%	Understand and use the concept of expected loss, or risk, such as in playing unfair game or buying an insurance policy.	Use simulation to solve realistic probability problems (e.g., the distribution of the number of donors who must come to a blood bank in order to see three with type A blood). C-Plus 1% Glencoe .4%

Continued:

Standard: Data Analysis and Probability Strand: Probability Concepts and Applications

Summary of % of Lessons Aligned to Individual Probability Concepts and Applications Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
4)	After observing the proportion of elements in a small population that have certain characteristic (e.g., the proportion of girls in the classroom), connect this proportion to the probability of obtaining this characteristic in an element randomly sampled from the same population (e.g., will selecting a girl be more likely than selecting a boy?). C-Plus .5%	Demonstrate the concept of random variation through the repetition of a sample or an experiment. For each experiment (or sample), estimate the probabilities for simple outcomes and compare the estimates across several experiments (or samples). C-Plus 2.1% M Conn .6%	Investigate the behavior of empirical probabilities of events through simulation using dice, spinners, random number tables, or random number generators, such as those found on calculators or computers. C-Plus 2.7% M Conn 1.3%	Use two-way tables to find probabilities of compound events. Glencoe .4%	Simulate probability distributions for discrete random variables, such as the number of boxes of cereal to be purchased in order to find a specific prize. C-Plus 2.7% Glencoe .4%	NA
5)	Use the language of probability to describe the likelihood of an event; compare empirical and theoretical probabilities for several random devices (e.g., spinners, dice) and for small populations. C-Plus 1.6% Glencoe .4% M Conn 1.3%	NA	Recognize that the theoretical probability represents the long-run chance of an event occurring, and recognize that empirical estimates of probabilities become more stable as more data is collected. C-Plus 2.1%	Be able to use the terms likely, equally likely, and unlikely to describe everyday events as well as the outcomes of an experiment or sample survey. C-Plus 2.1% Glencoe .4% M Conn 1.3%	NA	NA

Standard: Data Analysis and Probability ***Strand: Problem Solving***

Core-Plus

- 9.6% of Total Lessons addressed a Performance Expectation in this Strand
- 32% of the Problem Solving Performance Expectations addressed

Glencoe

- 0% of the Problem Solving Performance Expectations addressed

Math Connections

- 3.8% of Total Lessons addressed a Performance Expectation in this Strand
- 5% of the Problem Solving Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Problem Solving Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	When presented with a simple "research question" about a small population, identify an appropriate characteristic on which to collect categorical data. C-Plus .5%	When presented with a simple "research question" about a small population, identify an appropriate characteristic on which to collect numerical data.	Identify appropriate data, processes, and representations in problem situations involving bivariate numerical data and the concept of association in small populations. C-Plus .5%	Identify a problem that can be addressed through data collection and analysis of census or survey data.	Identify problems that can be addressed through collection and analysis of experimental data, and recognize factors that may affect the problem.	Investigate more complex problems through data collection. Understand the scope of the problem, and formulate questions that can be addressed through data collection.
2)	Decide methods for collecting, recording and summarizing data, and use the resulting data to answer the question of interest (e.g., if the research question is, "What is your favorite food?" students should decide what population they will observe, a definition for "food," and methods for recording and summarizing the data). C-Plus .5%	Decide methods for collecting, recording, and summarizing numerical data, and use the resulting data to answer the question of interest (e.g., if the research question is, "How big is a person's arm span?" students should decide on a small population to observe, how to measure arm span, and methods for recording and summarizing the data). M Conn 3.8%	Describe problem situations involving bivariate data, both in those requiring two numerical variables and in those requiring one numerical and one categorical variable, and identify appropriate problem-solving strategies for such situations.	Understand and describe how to quantify key components of a problem so that they can be addressed through data collection and analysis. C-Plus .5%	Identify collected variables as response or explanatory variables.	Distinguish among problems that may be best explored through a sample survey, a designed experiment, or an observational study.

Continued:

Standard: Data Analysis and Probability ***Strand: Problem Solving***

Summary of % of Lessons Aligned to Individual Problem Solving Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
3)	NA	NA	Solve problems involving bivariate data that require examining the association between two numerical characteristics of individuals in a small population or comparing two or more small populations on some characteristic. C-Plus 5.9%	Decide what population to study, what outcome to measure, how to measure outcome, and what factors may influence the outcome.	Formulate specific questions, and identify quantitative measures that may be used to provide answers.	Evaluate different possible factors involved in a problem and what quantitative information may be obtained from them. Decide what factors to measure and how to measure them.
4)	NA	NA	NA	Identify appropriate sources for gathering data from a population. Students may collect data directly or find sources that have already collected (e.g., federal statistical agencies, magazines, newspapers, Internet sites).	Design and carry out an experiment to answer the question.	Consider the advantages and disadvantages of different methods of measuring variables (e.g., face-to-face interviews versus paper-and-pencil questionnaires).
5)	NA	NA	NA	Collect and summarize relevant data, and propose a solution to the original problem. C-Plus .5%	Draw appropriate conclusions from the collected data. C-Plus 1%	Chose methods of data analysis appropriate for the variables collected and the methods of data collection.

Standard: Data Analysis and Probability ***Strand: Representations***

Core-Plus

- 35.8% of Total Lessons addressed a Performance Expectation in this Strand
- 79% of the Representations Performance Expectations addressed

Glencoe

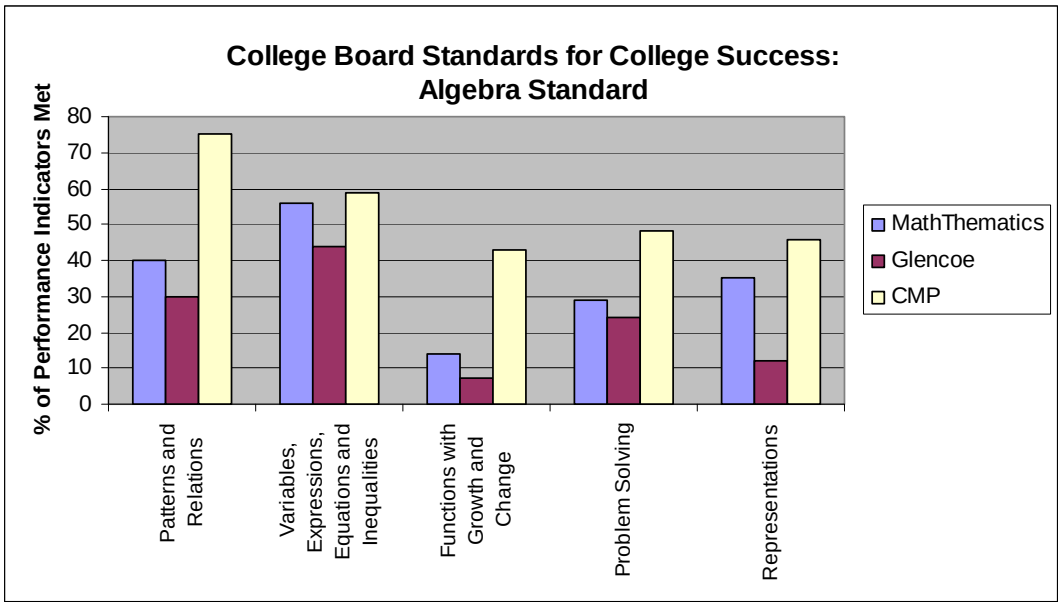
- 3.2% of Total Lessons addressed a Performance Expectation in this Strand
- 29% of the Representations Performance Expectations addressed

Math Connections

- 18.5% of Total Lessons addressed a Performance Expectation in this Strand
- 50% of the Representations Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Representations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Recognize and select appropriate representations for categorical data. M Conn .6%	Recognize and select appropriate representations, summary statistics, and plots for numerical data. C-Plus 1% M Conn 4.5%	Recognize and select appropriate tabular and graphical representations for bivariate numerical data from a single population. C-Plus .5%	Organize and present univariate data from a sample or a census in frequency distributions, tables, or graphs with appropriate scales, labels, and titles. C-Plus 3.7%	Display the study data in tables and graphs, including best-fit lines for scatterplot data to describe the trend in the data. C-Plus .7% Glencoe .4% M Conn .6%	Recognize the advantages of different graphical and tabular displays of the same data. C-Plus 1.6%
2)	Select, make, and interpret tables and graphs, such as frequency tables, pictographs, bar charts, and pie charts, for categorical data. C-Plus 4.3% M Conn .6%	Select, make, and interpret tables and graphs, such as dot plots, stem plants, box plots, frequency tables, and histograms, for numerical data. C-Plus 9.1% Glencoe 1.4% M Conn 3.8%	Recognize and select appropriate tabular and graphical representations for comparing univariate numerical data from several populations.	Display bivariate data from a sample or a census in two-way tables of counts (categorical data) or scatterplots (numerical data) with appropriate scales and labels. C-Plus 1%	Describe the sampling variability in a sample statistic through the use of simulation to construct the sampling distribution of the statistic.	Understand that different scales and representations may reveal different characteristics of the data. C-Plus 1%
3)	NA	Compute numerical summaries of center, such as mean, median, and mode, and summaries of spread, such as range. C-Plus 2.7% Glencoe .7% M Conn 4.5%	Select, make, and interpret tabular and graphical representations for bivariate data such as scatterplots and parallel box plots. C-Plus 7% Glencoe .7% M Conn 3.8%	NA	NA	NA

Algebra Strand/Middle School Analysis



This graph depicts the percent of Level I-VI Performance Expectations addressed for each Strand of the Algebra Standard. For all Strands, the Connected Mathematics Project (CMP) program addressed a higher percentage of Performance Expectations than Glencoe and MathThematics. Additional detail on percentage of lessons aligned to each of the Performance Expectations within a Strand follows below.

In comparing the Level I – VI Algebra Performance Expectations to the Course Performance Expectations, Level I is considered pre-algebra and Levels II and III, algebra. When aligned to an Expectation, MathThematics and CMP seem to be at or above the College Board grade level and Glencoe seems to be at College Board grade level

Standard: Algebra ***Strand: Patterns and Relations***

MathThematics

- 6.4% of Total Lessons addressed a Performance Expectation in this Strand
- 40% of the Patterns and Relations Performance Expectations addressed

Glencoe

- 6.5% of Total Lessons addressed a Performance Expectation in this Strand
- 30% of the Patterns and Relations Performance Expectations addressed

CMP

- 15.3% of Total Lessons addressed a Performance Expectation in this Strand
- 75% of the Patterns and Relations Performance Expectations addressed

Summary of % of Lessons Aligned to Individual Patterns and Relations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Recognize and describe linear relationships and patterns, and distinguish between linear and nonlinear patterns. M Them .3% Glencoe .7% CMP 1%	Recognize and describe multistep linear relationships and patterns. MThem 1% Glencoe .4% CMP .7%	Recognize and describe absolute value, piecewise-defined, and quadratic patterns and relationships. MThem .6% Glencoe .4% CMP 1.3%	Recognize and describe rational, radical, and polynomial patterns and relationships. CMP .3%	Recognize and describe exponential, logarithmic, and recursive patterns and relationships. CMP .7%	Recognize, create, and describe trigonometric, polar, and parametric patterns and relationships.
2)	Create and extend simple linear patterns. MThem 1.3% Glencoe 1.8% CMP 1.3%	Recognize a rate as a comparison of the change in one quantity relative to a one-unit change in a second quantity. MThem .6% Glencoe 2.5% CMP 2.6%	Create and extend absolute value, piecewise-defined, and quadratic patterns and relationships. MThem .3% CMP .7%	Recognize the relationship between a function and its inverse, verbally, numerically, graphically, and symbolically. CMP .7%	Create and extend exponential, logarithmic, and recursive patterns and relationships, and their inverses. CMP .3%	Create and extend trigonometric, polar, and parametric patterns and relationships, and their inverses.
3)	Generalize simple linear patterns with a verbal or symbolic rule. MThem 1.9% Glencoe .7% CMP 2.9%	Create and extend multistep linear patterns. CMP .3%	Generalize absolute value, piecewise-defined, and quadratic patterns and relationships with a verbal or symbolic rule. CMP 1	Create and extend rational, radical, and polynomial patterns and relationships, and their inverses.	Generalize exponential, logarithmic, and recursive patterns and relationships with a verbal or symbolic rule. CMP .7%	Generalize trigonometric, polar, and parametric patterns and relationships with a verbal or symbolic rule.
4)	NA	Generalize multistep linear patterns with a verbal or symbolic rule. MThem .3% CMP 1%	NA	Generalize rational, radical, and polynomial patterns and relationships with a verbal or symbolic rule.	NA	NA

Standard: Algebra Strand: Variables, Expressions, Equations and Inequalities

MathThematics

- 19.2% of Total Lessons addressed a Performance Expectation in this Strand
- 56% of the Variables, Expressions, Equations and Inequalities Performance Expectations addressed

Glencoe

- 23.8% of Total Lessons addressed a Performance Expectation in this Strand
- 44% of the Variables, Expressions, Equations and Inequalities Performance Expectations addressed

CMP

- 17.9% of Total Lessons addressed a Performance Expectation in this Strand
- 59% of the Variables, Expressions, Equations and Inequalities Expectations addressed
-

Summary of % of Lessons Aligned to Individual Variables, Expressions, Equations and Inequalities Performance Expectations (no Level 6 alignment found)					
PE	Level 1	Level 2	Level 3	Level 4	Level 5
1)	Identify the constants and variable terms in simple linear relationships and patterns, and identify the independent and dependent variables in linear relationships. Glencoe .4%	Create linear expressions, equations, and inequalities that represent algebraic situations, including systems of equations and inequalities. MThem 1.3% Glencoe .7% CMP 1.6%	Create absolute value, piecewise- defined, and quadratic expressions and equations that represent algebraic situations. MThem 1% CMP 1.3%	Create rational, radical, and polynomial expressions and equations that represent algebraic situations. MThem .3%	Create exponential, logarithmic, and recursive expressions and equations that represent algebraic situations. CMP .3%
2)	Set up and solve proportions involving whole numbers. MThem 3.2 % Glencoe 4.7% CMP .7%	Evaluate and simplify linear expressions, linear equations, linear inequalities, and systems of linear equations and inequalities. MThem 1% Glencoe .7% CMP 1.3%	Evaluate absolute value, piecewise-defined, and quadratic expressions. MThem .3% Glencoe .4% CMP .3%	Simplify and evaluate rational, radical, and polynomial expressions and equations. MThem .3%	Graph and interpret the graphs of exponential, logarithmic, and recursive functions. CMP .7%
3)	Create linear expressions and equations that represent algebraic situations. MThem .6% Glencoe 1.1% CMP 2.3%	Solve linear equations, linear inequalities, and systems of linear equations and inequalities. MThem 2.6% Glencoe 3.2% CMP .7%	Simplify absolute value, piecewise-defined, and quadratic expressions and equations. MThem .6% Glencoe 1.4% CMP .3%	Solve rational, radical, and polynomial equations and systems of these equations.	Simplify and evaluate exponential, logarithmic, and recursive expressions and equations.

Continued:

Standard: Algebra ***Strand: Variables, Expressions, Equations and Inequalities***

Summary of % of Lessons Aligned to Individual Variables, Expressions, Equations and Inequalities Performance Expectations (no Level 6 alignment found)					
PE	Level 1	Level 2	Level 3	Level 4	Level 5
4)	Evaluate and simplify linear expressions. MThem 1.9% Glencoe 1.4% CMP 1%	Identify the constants and variable terms in linear equations, linear inequalities, and systems of linear equations and inequalities, including independent and dependent variables. Glencoe .4%	Solve absolute value, piecewise- defined, and quadratic equations and systems of these equations. MThem 1% CMP .3%	Identify the constants and variable terms in rational, radical, and polynomial functions.	Solve exponential, logarithmic, and recursive equations and systems of these equations.
5)	Solve one-step linear equations. MThem 1% Glencoe 4.3% CMP 2%	Apply the commutative, associative, and distributive properties to simplify and solve algebraic expressions and equations. MThem 3.2% Glencoe 5.4% CMP 4.2%	Identify the constants and variable terms in absolute value, piecewise- defined, and quadratic equations and inequalities, including the independent and dependent variables. CMP .3%	Use properties of inverses to solve algebraic problems. MThem 1% CMP .7%	NA

Standard: Algebra ***Strand: Functions with Growth and Change***

MathThematics

- 1% of Total Lessons addressed a Performance Expectation in this Strand
- 14% of the Functions with Growth and Change Performance Expectations addressed

Glencoe

- .4% of Total Lessons addressed a Performance Expectation in this Strand
- 7% of the Functions with Growth and Change Performance Expectations addressed

CMP

- 4.2% of Total Lessons addressed a Performance Expectation in this Strand
- 43% of the Functions with Growth and Change Expectations addressed

Summary of % of Lessons Aligned to Individual Functions with Growth and Change Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	NA	NA	NA	Solve problems involving rational, radical, and polynomial functions.	Identify the constants and variable terms in exponential, logarithmic, and recursive functions. CMP .3%	Identify the constants and variable terms in trigonometric, polar, and parametric functions.
2)	NA	NA	NA	Determine whether a given relation is a function or not based on its definition or on its graphical representation. CMP 1%	Understand the relationship between exponential and logarithmic functions.	Perform the composition of linear and trigonometric functions.
3)	NA	NA	NA	Graph and interpret the graphs of absolute value, piecewise-defined, and quadratic functions, in terms of zeros, intercepts, and overall qualitative behavior. CMP 1%	Demonstrate an understanding of recursively defined functions.	Demonstrate an understanding of trigonometric, inverse trigonometric, polar, and parametric functions.
4)	NA	NA	NA	Relate the changes in functional values to changes in the domain values. MThem .6% CMP .7%	Understand exponential, logarithmic, and recursive functions, and sequences and series.	Find the sum of terms in an arithmetic series and in finite and infinite geometric series.
5)	NA	NA	NA	Demonstrate an understanding of the composition of linear and quadratic functions. MThem .3% Glencoe .4% CMP 1%	Demonstrate an understanding of the composition of linear, exponential, and logarithmic functions. CMP .3%	NA

Standard: Algebra ***Strand: Problem Solving***

MathThematics

- 4.2% of Total Lessons addressed a Performance Expectation in this Strand
- 29% of the Problem Solving Performance Expectations addressed

Glencoe

- 7.2% of Total Lessons addressed a Performance Expectation in this Strand
- 24% of the Problem Solving Performance Expectations addressed

CMP

- 10.4% of Total Lessons addressed a Performance Expectation in this Strand
- 48% of the Problem Solving Expectations addressed

Summary of % of Lessons Aligned to Individual Problem Solving Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Describe problem situations involving simple linear relationships and patterns, and identify appropriate problem-solving strategies for such situations. CMP .7%	Describe problem situations involving linear equations, linear inequalities, and systems of linear equations and inequalities, and identify appropriate problem-solving strategies for such situations.	Describe problem situations involving absolute value, piecewise-defined, and quadratic equations, and identify appropriate problem-solving strategies for such situations. MThem .3%	Describe problem situations involving rational, radical, and polynomial functions, and identify appropriate problem-solving strategies for such situations.	Describe problem situations involving exponential, logarithmic, and recursive functions, and identify appropriate problem-solving strategies for such situations. CMP .3%	Describe problem situations involving trigonometric, inverse trigonometric, polar, and parametric functions, and identify appropriate problem-solving strategies for such situations.
2)	Solve problems involving simple linear relationships and patterns. MThem .3% Glencoe 1.8% CMP 1.6%	Solve problems involving rates, showing the relationships in algebraic and geometric forms. MThem 1.9% Glencoe .4% CMP 1.3%	Solve problems involving absolute value, piecewise-defined, and quadratic equations.	Apply the properties of matrices to solving systems of equations.	Solve problems involving exponential, logarithmic, and recursive functions and sequences and series. CMP .7%	Solve problems involving the composition of linear and trigonometric functions.
3)	Apply order of operations to algebraic expressions in problem solving. MThem .3% Glencoe 3.2% CMP 1.6%	Solve problems involving linear equations, linear inequalities, and systems of linear equations and inequalities. MThem .6% Glencoe 1.1% CMP 2.3%	NA	Solve problems involving the composition of linear and quadratic functions. CMP .7%	Solve problems involving the composition of linear, exponential, and logarithmic functions. CMP .3%	Solve problems involving trigonometric, inverse trigonometric, polar, and parametric functions.
4)	NA	Solve multistep problems involving linear equations and inequalities. MThem .6% Glencoe .7% CMP .3%	NA	Solve problems involving linear programming and matrices.	Apply the properties of logarithms and exponents and the equivalence between $y=a^x$ and $x = \log_a y$ to the solution of problems involving logarithmic and exponential equations.	Solve problems involving the sum of terms in an arithmetic series and in finite and infinite geometric series.

Standard: Algebra ***Strand: Representations***

MathThematics

- 5.4% of Total Lessons addressed a Performance Expectation in this Strand
- 35% of the Representations Performance Expectations addressed

Glencoe

- 3.2% of Total Lessons addressed a Performance Expectation in this Strand
- 12% of the Representations Performance Expectations addressed

CMP

- 11.4% of Total Lessons addressed a Performance Expectation in this Strand
- 46% of the Representations Expectations addressed

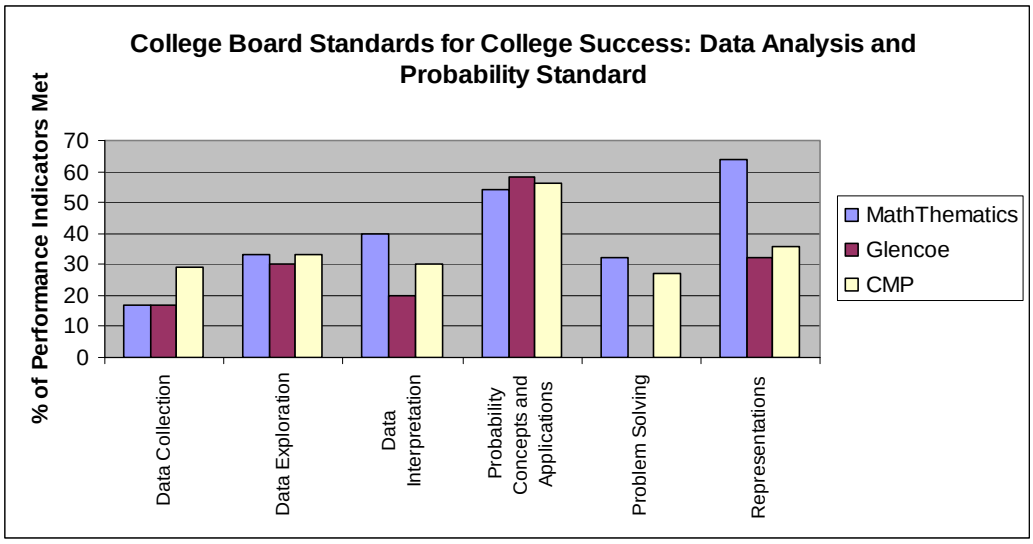
Summary of % of Lessons Aligned to Individual Representations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Recognize or create appropriate representations of simple linear relationships and patterns, including verbal, tabular, graphical, and symbolic representations. MThem .3% Glencoe .7% CMP 3.3%	Recognize and create appropriate representations of linear equations, linear inequalities, and systems of linear equations and inequalities, including verbal, tabular, graphical, and symbolic representations. MThem 1.3% Glencoe 2.2%	Recognize and create appropriate representations of absolute value, piecewise-defined, and quadratic functions. MThem .6% Glencoe .4% CMP 1 %	Recognize and create appropriate representations, including matrices, of rational, radical, and polynomial functions, including verbal, tabular, graphical, and symbolic representations.	Recognize and create appropriate representations, including matrices, of exponential, logarithmic, and recursive functions, including verbal, tabular, graphical, and symbolic representations. CMP .3%	Recognize and create appropriate representations of trigonometric inverse trigonometric, polar, and parametric functions, including verbal, tabular, graphical, and symbolic representations.
2)	Describe the characteristics and defend the appropriateness of alternative representations of simple linear relationships and patterns. MThem .3% CMP .7%	Describe the characteristics and defend the appropriateness of alternative representations of linear equations, linear inequalities and systems of equations and inequalities. MThem .6%	Describe the characteristics and defend the appropriateness of alternative representations of absolute value, piecewise-defined and quadratic functions.	Describe the characteristics and defend the appropriateness of alternative representations of rational, radical, and polynomial functions.	Describe the characteristics and defend the appropriateness of alternative representations of exponential, logarithmic, and recursive functions, and sequences and series.	Describe the characteristics and defend the appropriateness of alternative representations of trigonometric, inverse trigonometric, polar, and parametric functions.
3)	Identify and recognize equivalent verbal, tabular, graphical, and symbolic representations of simple linear relationships and patterns. CMP 1.3%	Identify and recognize equivalent representations, including verbal, tabular, graphical, and symbolic representations, of linear equations and inequalities and systems of linear equations. MThem .6% CMP .3%	Recognize, apply, and translate among equivalent representations of absolute value, piecewise-defined, and quadratic functions, including verbal, graphical, and symbolic representations. MThem .3% CMP 1%	Identify, recognize, and apply equivalent representations of rational, radical, and polynomial functions, including verbal, tabular, graphical, and symbolic representations.	Identify and apply equivalent representations of exponential, logarithmic, and recursive functions, including verbal, tabular, graphical, and symbolic representations. CMP .7%	Identify and apply equivalent representations of trigonometric, inverse trigonometric, polar, and parametric functions, including verbal, tabular, graphical, and symbolic representations.

Continued:

Standard: Algebra ***Strand: Representations***

Summary of % of Lessons Aligned to Individual Representations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
4)	Use and translate among equivalent representations of simple linear relationships and patterns. CMP .7%	Use and translate among equivalent representations of linear equations, linear inequalities, and systems of linear equations and inequalities. MThem 1% CMP .3%	Compare and contrast equivalent representations of absolute value, piecewise-defined, and quadratic functions.	Compare and contrast equivalent representations of rational, radical, and polynomial functions.	Compare and contrast equivalent representations of exponential, logarithmic, and recursive functions, and sequences and series.	Compare and contrast equivalent representations of trigonometric, inverse trigonometric, polar, and parametric functions.
5)	Compare and contrast equivalent representations of simple linear relationships and patterns. CMP 2%	Compare and contrast equivalent representations of linear equations, linear inequalities, and systems of linear equations and inequalities. MThem .3% CMP .7%	NA	NA	NA	NA

Data Analysis and Probability Standard/Middle School Analysis



This graph depicts the percent of Level I-VI Performance Expectations addressed for each Strand of the Data Analysis and Probability Standard. In three of the six Strands, MathThematics aligned with a higher percentage of performance Expectations than Glencoe and CMP. Additional detail on percentage of lessons aligned to each of the Performance Expectations within a strand follows below.

In comparing the Level I – VI Data Analysis and Probability Performance Expectations to the Course Performance Expectations, the majority of Level I II and III expectations are considered middle level content. When aligned to an Expectation, MathThematics and CMP seem to be at or above the College Board grade level and Glencoe seems to be at College Board grade level

Standard: Data Analysis and Probability ***Strand: Data Collection***

MathThematics

- 2.6% of Total Lessons addressed a Performance Expectation in this Strand
- 17% of the Data Collection Performance Expectations addressed

Glencoe

- 1.8% of Total Lessons addressed a Performance Expectation in this Strand
- 17% of the Data Collection Performance Expectations addressed

CMP

- 4.6% of Total Lessons addressed a Performance Expectation in this Strand
- 29% of the Data Collection Expectations addressed

Summary of % of Lessons Aligned to Individual Data Collection Performance Expectations (no Level 5 or 6 alignment found)				
PE	Level 1	Level 2	Level 3	Level 4
1)	Describe data collection processes for gathering univariate categorical data on a small population.	Describe data collection processes for gathering univariate numerical data on a small population. CMP .3%	Describe data collection processes for gathering bivariate data (two variables for each case). CMP .3%	Describe both a census and a simple random sample from a population. Recognize and describe the differences between population parameters and sample statistics.
2)	Distinguish between "Cases," the units that are measured (e.g., an individual student), and "Variables," the characteristic that is measured (e.g., favorite food).	Describe a method for measuring the variable of interest when gathering univariate numerical data on a small population. Glencoe .4%	Recognize three types of bivariate data: the variables may both be categorical, both numerical, or one numerical and one categorical.	Describe techniques for drawing a simple random sample (e.g., giving every unit in the population an ID number, writing the numbers on identical slips of paper, and drawing slips of paper at random from a bowl to select the units of the sample). CMP .3%
3)	Collect univariate categorical data on a small population by classifying observations into one of several categories (e.g., favorite foods of the students in a given classroom).	Collect univariate numerical data on a small population (e.g., statistical investigations of characteristics, such as height or arm span of students in a given classroom or the number of books owned by boys and girls in the class). MThem .6% Glencoe .7% CMP 1.6%	Recognize two settings for bivariate analysis: two measurements on each subject in a single small population, or one measurement on each subject in two or more small populations (e.g., statistical investigations may be made of such situations as arm span versus height for students in a single classroom or heights of students in two or more classrooms). MThem .3% CMP .7%	Describe types of nonrandom sampling, such as convenience (haphazard) sampling and judgment sampling.
4)	NA	Organize collected data so that it can be effectively displayed. MThem .6% Glencoe .4% CMP 1%	Collect all three types of bivariate data.	Draw a small simple random sample from a population, collect numerical and/or categorical data on sampled subjects, and organize the collected sample data so that it can be effectively displayed in tables or graphs.
5)	NA	NA	Organize the collected data so that it can be effectively displayed in tables or graphs. MThem 1% Glencoe .4% CMP .3%	NA

Standard: Data Analysis and Probability ***Strand: Data Exploration***

MathThematics

- 6.4% of Total Lessons addressed a Performance Expectation in this Strand
- 33% of the Data Exploration Performance Expectations addressed

Glencoe

- 7.9% of Total Lessons addressed a Performance Expectation in this Strand
- 30% of the Data Exploration Performance Expectations addressed

CMP

- 4.6% of Total Lessons addressed a Performance Expectation in this Strand
- 33% of the Data Exploration Expectations addressed

Summary of % of Lessons Aligned to Individual Data Exploration Performance Expectations (no Level 6 alignment found)					
PE	Level 1	Level 2	Level 3	Level 4	Level 5
1)	Summarize the collected data, beginning with counts and then moving to proportions and percentages. Glencoe 1.4% CMP .3%	Identify and describe the processes and techniques for describing the distribution of values in a univariate numerical data set, including the use of graphical displays such as dot plots, stem plots, and histograms. MThem 2.2% Glencoe .7% CMP .3%	Identify, describe, and construct appropriate displays for studying the association between two variables: two-way tables for categorical versus categorical; parallel box plots (or other appropriate comparative graphs) for numerical versus categorical; and scatterplots for numerical versus numerical. MThem .6% CMP .7%	Recognize that summary numbers, such as center and spread computed from a random sample, are likely to differ from the same summary numbers computed from a census of the same population. CMP .3%	Plot experimental data, and look for patterns in the plots. For example, check for an association between two numerical variables by making a scatterplot and examining it for patterns. MThem .6%
2)	Recognize that the categories might be ordered and that the category labeled "small size" (e.g., T-shirt size) may have the largest frequency of use. Glencoe .4%	Describe the overall shape of the distribution, including clumps, bumps, and gaps in the data, extremes, and range. MThem .3% Glencoe .7% CMP .3%	Recognize and describe the similarities and differences among the three types of displays: for example, outliers affect the degree of association exhibited in all three types; tables of categorical data and scatterplots each can show trends. MThem .6% CMP .3%	Use simulation to explore the variability of characteristics of random samples, such as shape, center, and spread in samples drawn from a known population, and to study the sampling distributions of means and proportions.	Compute a correlation between two numerical variables as a measure of strength of association.

Continued:

Standard: Data Analysis and Probability ***Strand: Data Exploration***

Summary of % of Lessons Aligned to Individual Data Exploration Performance Expectations (no Level 6 alignment found)					
PE	Level 1	Level 2	Level 3	Level 4	Level 5
3)	Describe the distribution of values in a univariate categorical data set through the use of graphical displays, such as bar charts and pie charts. MThem .6% Glencoe 1.4% CMP .3%	Recognize, be bale to compute, and interpret measures of center, such as the mean, the median, and the mode, and simple measures of spread, such as range and interquartile range. MThem .6% Glencoe 1.8% CMP 1.3%	NA	Use simulation to explore what kinds of sample results might be expected under various assumptions about the population (e.g., if 35 percent of the students in the school like a particular school policy, what would the distribution of sample proportions look like for samples of 25?).	Use a lest-squares regression line or a median-fit line to describe the type of association shown in a scatterplot.
4)	Compare proportions/percentages in each category of a univariate categorical data set, and determine ordinal rankings such as most and least. MThem .3% Glencoe .7%	Relate measures of center and spread to graphical representations of data. MThem .3% Glencoe .4% CMP .7%	NA	Compare results from random and non-random samples of the same population.	Use a simple randomization test to compare two means in a designed experiment in which treatments were randomly assigned to experimental units.

Standard: Data Analysis and Probability ***Strand: Data Interpretation***

MathThematics

- 2.6% of Total Lessons addressed a Performance Expectation in this Strand
- 40% of the Data Interpretation Performance Expectations addressed

Glencoe

- 3.2% of Total Lessons addressed a Performance Expectation in this Strand
- 20% of the Data Interpretation Performance Expectations addressed

CMP

- 2.9% of Total Lessons addressed a Performance Expectation in this Strand
- 30% of the Data Interpretation Expectations addressed

Summary of % of Lessons Aligned to Individual Data Interpretation Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Use the results of a categorical data collection to answer the question that motivated the gathering of data. Glencoe 1.1%	Recognize the difference between numerical and categorical data: for example, that a count, in the context of numerical data, represents the measurement on a single person and not a summary statistic for the class. Glencoe .4% CMP .3%	Discuss the association between two numerical characteristics measured on the same population. CMP .7%	Compare the advantages and disadvantages of taking a census, drawing a random sample, or using a nonrandom sample. MThem .3% CMP .3%	Distinguish between associations and a casual relationship between variables.	Recognize that sample-to-sample variability must be taken into account when making inferences from sample data to a population.
2)	Critique an analysis of univariate categorical data, such as a bar graph or a pie chart found in a newspaper of magazine, with regard to the data display used and the reasonableness of the conclusions drawn. MThem .3% Glencoe .7%	Use the results of a numerical data collection to answer a question of interest, and discuss what results mean in the context of the question of interest. MThem .3% Glencoe .7% CMP 1%	Compare and contrast the distributions of some numerical characteristic measured in two or more populations. MThem .3%	2Recognize when it is appropriate to draw conclusions about a population from sample data, and recognize the importance of sample-to-sample variability in interpreting results from sample data. CMP .3%	Interpret the correlations between two numerical variables. MThem .3%	Explain how confidence intervals can be used to summarize sample survey or experimental results, and interpret confidence intervals and confidence levels.

Continued:

Standard: Data Analysis and Probability ***Strand: Data Interpretation***

Summary of % of Lessons Aligned to Individual Data Interpretation Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
3)	NA	Critique an analysis of numerical data, such as a line graph or a histogram found in an article or book, particularly the reasonableness of the conclusions drawn.	Critique an analysis of bivariate data, such as a scatterplot found in an article or book, particularly the reasonableness of the conclusions drawn.	Identify possible sources of bias in sample data: for example, recognize factors that may affect sample results, such as the way survey questions are worded or the method used to draw the sample. MThem .3% Glencoe .4% CMP .3%	Interpret the slope and intercept of a regression line.	Recognize the effects of a linear transformation on the center, shape, and spread of distribution.
4)	NA	NA	NA	Critique an analysis of sample data, such as the appropriateness of the conclusions drawn from the sample because of the way the sample was collected from the population. MThem .3%	Interpret the results of randomization test of means.	Recognize that data scales are chosen for convenience of analyzing and presenting information. MThem .3%

Standard: Data Analysis and Probability Strand: Probability Concepts and Applications

MathThematics

- 12.8% of Total Lessons addressed a Performance Expectation in this Strand
- 54% of the Probability Concepts and Applications Performance Expectations addressed

Glencoe

- 11.9% of Total Lessons addressed a Performance Expectation in this Strand
- 58% of the Probability Concepts and Applications Performance Expectations addressed

CMP

- 13.7% of Total Lessons addressed a Performance Expectation in this Strand
- 46% of the Probability Concepts and Applications Expectations addressed

Summary of % of Lessons Aligned to Individual Probability Concepts and Applications Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Conduct experiments when known structure (e.g., with coins, dice, spinners, or cards) to estimate the likelihood of simple outcomes. MThem 1% Glencoe .4% CMP 1.3%	Collect sample or experimental data to estimate probabilities of outcomes for which the theoretical probability is unknown, (e.g., the probability that a randomly selected person from the school will have a pet dog). Summarize the data in a table or graph. MThem .3% Glencoe .7% CMP .3%	Describe the set of possible outcomes (sample space) of a random experiment. Define events as subsets of the set of possible outcomes (e.g., in the experiment of tossing a die, describe the event "the outcome is even" as a subset of the possible outcomes). Recognize complementary and mutually exclusive events. MThem 1% Glencoe .4%	Distinguish between independent and dependent events, and understand the basic idea of conditional probability. MThem .6% Glencoe .7%	Understand the notion of a random variable and its probability distribution, such as that of the number of heads when three coins are tossed.	Use combinations or permutations to count outcomes in more complex sample spaces. Glencoe .7%
2)	Relate the estimate (empirical probability) to the structure of the device (theoretical probability), such as the sizes of the colored regions on the spinner or the number of red cards in the deck. Glencoe .4%	Recognize that if the estimated probability is to be reliable, the collected data must satisfy certain conditions: for example, the data must form a representative sample of the population. Glencoe .7%	Assign probabilities, either theoretical or empirical, to events. MThem 2.6% Glencoe .7% CMP 1%	Understand and use the additive law of probability, and understand and use the multiplicative law of probability. MThem .6% Glencoe .4%	Compute expected values for experimental situations involving counts (e.g., the expected number of heads in 10 tosses of a fair coin).	Develop and use the binomial probability distribution

Continued:
Standard: Data Analysis and Probability Strand: Probability Concepts and Applications

Summary of % of Lessons Aligned to Individual Probability Concepts and Applications Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
3)	Identify the probability of an event as a number between 0 and 1, and compare the likelihood of different outcomes (more likely, less likely); for example, compare outcomes from spinners with differently sized regions. MThem 1.3% Glencoe .4% CMP 1.3%	Express a probability as a fraction, a decimal, or a percent. MThem 2.2% Glencoe 4% CMP 2.9%	Use Venn diagrams to show relationships among events and to help establish probabilities of these events. MThem .3%	Use tree diagrams to find probabilities of compound events. MThem 1.3% Glencoe .7% CMP 2%	Understand and use the concept of expected loss, or risk, such as in playing unfair game or buying an insurance policy. MThem .3% CMP 1%	Use simulation to solve realistic probability problems (e.g., the distribution of the number of donors who must come to a blood bank in order to see three with type A blood).
4)	After observing the proportion of elements in a small population that have certain characteristic (e.g., the proportion of girls in the classroom), connect this proportion to the probability of obtaining this characteristic in an element randomly sampled from the same population (e.g., will selecting a girl be more likely than selecting a boy?). CMP .3%	Demonstrate the concept of random variation through the repetition of a sample or an experiment. For each experiment (or sample), estimate the probabilities for simple outcomes and compare the estimates across several experiments (or samples). Glencoe .4% CMP .3%	Investigate the behavior of empirical probabilities of events through simulation using dice, spinners, random number tables, or random number generators, such as those found on calculators or computers. MThem .3% Glencoe .7% CMP .7%	Use two-way tables to find probabilities of compound events.	Simulate probability distributions for discrete random variables, such as the number of boxes of cereal to be purchased in order to find a specific prize.	NA
5)	Use the language of probability to describe the likelihood of an event; compare empirical and theoretical probabilities for several random devices (e.g., spinners, dice) and for small populations. MThem .6%	NA	Recognize that the theoretical probability represents the long-run chance of an event occurring, and recognize that empirical estimates of probabilities become more stable as more data is collected. CMP 1%	Be able to use the terms likely, equally likely, and unlikely to describe everyday events as well as the outcomes of an experiment or sample survey. MThem .3% Glencoe .4% CMP 1%	NA	NA

Standard: Data Analysis and Probability ***Strand: Problem Solving***

MathThematics

- 2.6% of Total Lessons addressed a Performance Expectation in this Strand
- 32% of the Problem Solving Performance Expectations addressed

Glencoe

- 0% of Total Lessons addressed a Performance Expectation in this Strand
- 0% of the Problem Solving Performance Expectations addressed

CMP

- 2.3% of Total Lessons addressed a Performance Expectation in this Strand
- 27% of the Problem Solving Expectations addressed

Summary of % of Lessons Aligned to Individual Problem Solving Performance Expectations (no Level 5 or 6 alignment found)				
PE	Level 1	Level 2	Level 3	Level 4
1)	When presented with a simple "research question" about a small population, identify an appropriate characteristic on which to collect categorical data. CMP .3%	When presented with a simple "research question" about a small population, identify an appropriate characteristic on which to collect numerical data.	Identify appropriate data, processes, and representations in problem situations involving bivariate numerical data and the concept of association in small populations. MThem .3% CMP .3%	Identify a problem that can be addressed through data collection and analysis of census or survey data. MThem .3%
2)	Decide methods for collecting, recording and summarizing data, and use the resulting data to answer the question of interest (e.g., if the research question is, "What is your favorite food?" students should decide what population they will observe, a definition for "food," and methods for recording and summarizing the data). CMP .3%	Decide methods for collecting, recording, and summarizing numerical data, and use the resulting data to answer the question of interest (e.g., if the research question is, "How big is a person's arm span?" students should decide on a small population to observe, how to measure arm span, and methods for recording and summarizing the data). MThem .3% CMP .3%	Describe problem situations involving bivariate data, both in those requiring two numerical variables and in those requiring one numerical and one categorical variable, and identify appropriate problem-solving strategies for such situations.	Understand and describe how to quantify key components of a problem so that they can be addressed through data collection and analysis.
3)	NA	NA	Solve problems involving bivariate data that require examining the association between two numerical characteristics of individuals in a small population or comparing two or more small populations on some characteristic. MThem .6% CMP .3%	Decide what population to study, what outcome to measure, how to measure outcome, and what factors may influence the outcome. MThem .3%
4)	NA	NA	NA	Identify appropriate sources for gathering data from a population. Students may collect data directly or find sources that have already collected (e.g., federal statistical agencies, magazines, newspapers, Internet sites). MThem .3%
5)	NA	NA	NA	Collect and summarize relevant data, and propose a solution to the original problem. MThem .3% CMP .3%

Standard: Data Analysis and Probability ***Strand: Representations***

MathThematics

- 11.2% of Total Lessons addressed a Performance Expectation in this Strand
- 64% of the Representations Performance Expectations addressed

Glencoe

- 11.9% of Total Lessons addressed a Performance Expectation in this Strand
- 32% of the Representations Performance Expectations addressed

CMP

- 6.8% of Total Lessons addressed a Performance Expectation in this Strand
- 36% of the Representations Expectations addressed

Summary of % of Lessons Aligned to Individual Representations Performance Expectations						
PE	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
1)	Recognize and select appropriate representations for categorical data.	Recognize and select appropriate representations, summary statistics, and plots for numerical data. MThem .3% Glencoe .7% CMP .3%	Recognize and select appropriate tabular and graphical representations for bivariate numerical data from a single population. Glencoe .4% CMP .3%	Organize and present univariate data from a sample or a census in frequency distributions, tables, or graphs with appropriate scales, labels, and titles.	Display the study data in tables and graphs, including best-fit lines for scatterplot data to describe the trend in the data. MThem .6%	Recognize the advantages of different graphical and tabular displays of the same data. MThem 1.3% CMP 1%
2)	Select, make, and interpret tables and graphs, such as frequency tables, pictographs, bar charts, and pie charts, for categorical data. MThem 1.6% Glencoe 1.4% CMP 1%	Select, make, and interpret tables and graphs, such as dot plots, stem plants, box plots, frequency tables, and histograms, for numerical data. MThem 2.6% Glencoe 5.8% CMP 2.3%	Recognize and select appropriate tabular and graphical representations for comparing univariate numerical data from several populations. MThem .3% CMP .3%	Display bivariate data from a sample or a census in two-way tables of counts (categorical data) or scatterplots (numerical data) with appropriate scales and labels.	Describe the sampling variability in a sample statistic through the use of simulation to construct the sampling distribution of the statistic.	Understand that different scales and representations may reveal different characteristics of the data. MThem 1.3% Glencoe 1.1%
3)	NA	Compute numerical summaries of center, such as mean, median, and mode, and summaries of spread, such as range. MThem 1.9% Glencoe 2.5% CMP .7%	Select, make, and interpret tabular and graphical representations for bivariate data such as scatterplots and parallel box plots. MThem 1.3% Glencoe 1.4% CMP 1%	NA	NA	NA

VIII. Application

Limitations of Initial Analysis (adapted from Project 2061):

The analysis was constrained in a number of ways that could affect the generalization of the findings and how they will be used.

First, there were constraints on the time and funding available for the analysis. As a result, MMSA could not include every textbook that is currently being utilized in Maine. For the same reasons, only the printed student and teacher editions were analyzed. The support materials, test banks, software, and other supplements that are available from some publishers were not included in the process.

A second limitation is connected to the alignment of goals and instructional criteria. The lesson analysis did not attempt a broad and comprehensive review of content but focused on the key concepts from the 5 content standards of the NCTM Standards. Even though the NCTM standards represent a national consensus, they may not align perfectly with the Maine Learning Results or the mathematics standards or framework of a given school district. The choice of instructional criteria, even though they are supported by research on mathematics teaching and learning, may not represent the philosophy of a particular district, school, or teacher. The criteria are intended to address features of materials that are most important for teaching and learning for the large majority of students and teachers.

The third limitation relates to the procedure itself. It is possible that a small number of lessons or activities that addressed area goals were overlooked, or that, occasionally, some instructional criteria were not given credit in a lesson. Also, while analysts made use of clarifications and indicators, they also used their judgement in assigning ratings to criteria. Fourth, although the analysts used the procedure carefully and objectively, individual biases from their experiences as teachers and professional developers might be present in the process. The indicators and criteria were designed to reduce this bias. (AAAS,2000).

All of these factors and others have the potential to contribute to uncertainties in the final analysis. However, the results and overall ratings can be used in identifying strengths and weaknesses of a program to help vertical teams identify areas to investigate further. As determined by the College Board, one of the primary goals of a vertical team is to improve academic performance for all students in earlier grades by introducing skills and concepts needed for success in AP and other challenging courses by identifying and developing Pre-AP strategies and vertically aligning curriculum towards the discipline-based AP standards. (College Board, 2004).

Limitations of Alignment Analysis to *College Board Standards for College Success*

This in-depth analysis to the College Board Standards for College Success will provide additional support in helping Maine teachers make informed decisions so that all students will have access to rigorous curriculum, encouraging secondary schools to better prepare more students for Advanced Placement courses.

The analysis of the three commonly used programs at both the middle and high school was conducted based on the Algebra and Data Analysis & Probability Standards of the College Board Standards for College Success. The Number and Operations, Geometry, and Measurement Standards were reviewed. Student knowledge of the performance expectations of these additional standards is important for a successful preparation for advanced level courses.

A second limitation to this analysis is in the level of alignment of a lesson to an individual performance expectation. Alignment occurred if the goal of the lesson met at least one part of the indicator and several indicators have multiple leaning targets.

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X: Appendix

Appendix A

A Numbers and Operations

A: Numbers and Number Sense and B: Computation

1. Understand numbers, ways of representing numbers, relationships among numbers, and number systems
   a. Numbers represent size or "how many" in a set, can be represented in multiple ways, are formed in a base-ten system with individual digits representing place-value
   b. Fractions, decimals, and percents can be used flexibly and each represents parts of a whole
   c. Whole numbers, integers, rational, irrational, complex and imaginary numbers represent classes of numbers that make up the real number system; understand their properties and structure.
   d. There are other number systems
   e. Graph numbers on number lines, identify coordinates
   f. Understand and use ratios and proportions to represent quantitative relationships

2 Understand meanings of operations and how they relate to one another
   a. Understand the effects of operations on magnitudes of quantities (i.e +,-,*,/, powers and roots)
   b. The commutative, associative, and distributive properties are important when operating with numbers
   c. Properties of one system do not always hold true in other systems
   d. Develop an understanding of permutations and combinations as counting techniques

3. Compute fluently and make reasonable estimates
   a. Develop strategies for operations on whole numbers that demonstrate both flexibility and fluency
   b. Develop and use reliable algorithms to solve arithmetic problems efficiently and accurately. Apply these algorithms to larger numbers and practiced for fluency.
   c. Develop strategies and algorithms for computing with integers, fractions, and decimals numbers that demonstrate both flexibility and fluency
   d. Develop strategies for computing with real numbers and have some basic proficiency with vectors and matrices
   e. Choose computational tools appropriately: mental math, paper and pencil strategies, estimation, and calculators

C Data Analysis and Probability

C: Data Analysis & Statistics and D: Probability

1 Select and use appropriate statistical methods to analyze data

a. Putting the individual data together draws attention to the set as a whole
b. There are tools to describe aggregated data. These summary statistics include measures of center (mean, median, mode) and spread (range, mean deviation, standards deviation)
c. Association and trends of bivariate data can be analyzed using tools such as scatterplots, fitted lines, regression models, and correlation.

2 Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

a. Collect real data to answer questions such as How many? How much? What kind? Which of these? Etc.
b. You can collect your own data or analyze data collected by others
c. There are various methods of data collection including surveys, observational studies, and experiments. Each of these methods serves a different purpose.
d. Data can be organized or ordered and this "picture" of the data provides information about the phenomena or question. Display type (i.e graphs, histograms, tables, boxplots, etc) should be chosen based on effectiveness and by analysis method.

3 Understand and apply basic concepts of probability

a. Understand the idea of chance and randomness by identifying events that are impossible, unlikely, likely, or certain
b. Theoretical outcomes can be determined for simple and compound events.(relative frequency)
c. Samples and simulations can help quantify the likelihood of an uncertain outcome
d. If an event is random and is repeated many times, then the distribution of outcomes forms a pattern
e. Understand the concepts of conditional probability, dependant and independent events.

4 Develop and evaluate inferences and predictions that are based on data

a. Defining an appropriate sample, collecting data from that sample, describing the sample are important in making reasonable inferences relating the sample and the population
b. Simulations can be used to investigate sampling distributions and make informal inferences

E Geometry

E: Geometry

1 Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

a. Observe, describe, identify a variety of shapes and their properties. Use properties to solve problems.
b. Inductive reasoning, deductive reasoning and formal proof techniques can be used in problem solving and to prove conjectures
c. Describe, represent, and investigate relationships within a geometric system (including congruence, similarity, and trigonometry)

2 Specify locations and describe spatial relationships using coordinate geometry and other representational systems

a. Relative position can be described using terms such as above, near, between, etc.
b. Rectangular grids can be used to locate position, measure the distance between two points, find slope, and analyze situations. In addition to the Cartesian coordinate system, other coordinate systems can be used to analyze situations
c. Visual representations such as number lines, arrays, Venn Diagrams, and vertex-edge graphs can be helpful in analyzing problems

3 Apply transformations and use symmetry to analyze mathematical situations

a. Slides/translations, flips/reflections, and turns/rotations are transformations. These transformations change a shape's position or orientation but not it's size or shape (isometries)
b. Dilations (magnifications and contractions) are transformations that may change a shape's position and orientation as well as it's size
c. Some shapes have line symmetry or rotational symmetry. It is possible to quantify aspects of symmetry
d. Transformations can be represented using matrix, vectors, or function notation

4 Use visualization, spatial reasoning, and geometric modeling to solve problems

a. Build, draw, and construct geometric objects including analyzing and drawing perspective views and moving between two and three dimensional objects and their representations
b. Describing, counting, and drawing attributes that can not be seen but can be inferred
c. Physically and mentally change the position, orientation, and size of objects in systematic ways
d. Use geometric models to solve problems in areas of mathematics and other disciplines

F Measurement

F: Measurement

1 Understand measurable attributes of objects and the units, systems, and processes of measurement

a. A measurable attribute is a characteristic of an object that can be quantified
b. Some attributes can be measured directly (length, weight, time, area, volume, temperature, angle measurement, etc.)
c. Standard units are necessary for consistency, different units are needed for different attributes
d. There are different standardized systems of measurement (customary, metric). Conversion between the systems
e. Changing an objects attribute may have an effect on certain measurements
f. Some attributes can be measured indirectly (speed, rate, density, etc)
g. All measurements are approximations so it is important to report an appropriate number of significant digits when computing with measurements

2 Apply appropriate techniques, tools, and formulas to determine measurements

a. Measurement techniques are strategies used to determine a measurement, such as counting, estimating, scaling, using formulas or tools, successive approximation, and upper & lower bounds.
b. Measurement tools are devices used to take measurements including rulers, scales, vessels, etc.
c. Units in measurement behave like variables under algebraic procedures
d. Select and apply techniques, tools, and formulas to accurately find length, distance, perimeter, area, volume, angle measurement, circumference, and surface area
e. Solve problems using scale factors, ratio, and proportion

H Algebra

H: Algebra and G: Patterns, Relations,

1 Understand patterns, relations, and functions
a. Patterns can be described by focusing on a term is obtained from the previous number i.e 2 ,4,6,….
b. Recursive patterns sequences appear naturally in many contexts (i.e Fibonacci Sequence)
c. Variables and algebraic symbols can be used to describe and extend patterns
d. A relationship can be represented with tables, graphs, and/or symbols
e. Understand the properties and characteristics of a variety of functions (and relations) and their graphs (including comparing, analyzing, and composing functions and their graphs)
f. Observed patterns need to be justified before relying on the results.

2 Represent and analyze mathematical situations and structures using algebraic symbols

a. A variable can be used as a placeholder for a specific number (i.e. 2 + __ = 5)
b. A variable can be used to denote a generalized arithmetic pattern (0 = 0 * m)
c. Write, solve, and explain expressions, equations, and inequalities in one or more variables. (including systems and linear programming)
d. The equal sign is a symbol of equivalence and balance
e. Symbolic algebra can be used to represent situations and to solve problems
f. Write and understand the meaning of equivalent forms of expressions, equations, inequalities, relations, and systems

3 Use mathematical models to represent and understand quantitative relationships

a. Objects, pictures, and symbols can be used to model situations that involve: addition, subtraction, multiplication, and division
b. Models can be used to make predictions or draw conclusions
c. Often data can be modeled with a specific class of functions

4 Analyze change in various contexts

a. Change can be described quantitatively (I grew 2 inches last summer) or qualitatively (I grew taller over the summer)
b. Graphs and tables can help see change visually
c. Change can be constant (i.e slope) or non constant and rates of change can be approximated and/or interpreted from graphical and numerical data

Category I: Identifying a Sense of Purpose

Part of planning a coherent curriculum involves deciding on its purposes and on what learning experiences will likely contribute to achieving those purposes. But while coherence from the designers' point of view is important, it may be inadequate to give students the same sense of what they are doing and why. This category includes criteria to determine whether the lesson/activity attempts to make its purposes clear and meaningful to the student and genuinely relates lessons to the unit purpose.

I.1 Conveying Unit Purpose: Does the lesson/activity convey an overall sense of purpose and direction that is understandable and motivating to students?

Clarification:
This criterion involves examining whether the lesson/activity begins with (or early on presents) an over-arching question or problem to be addressed by the unit (e.g., How can a graph help to make predictions?), or a representation of what will be learned (e.g., a concept map of the main ideas that will be explored), or otherwise identifies a purpose for the unit or chapter for the students (e.g., a clear statement of objectives, using know terms). The problem, question, representation or purpose provided by the lesson/activity should be explicit and comprehensible by the students, and it should be plausible that it would be interesting and/or motivating to them. A lesson/activity that begins with abstractions or phenomena outside students' range of perception, understanding, or knowledge does not adequately meet the criterion. However, a lesson/activity that starts with an unfamiliar but highly interesting phenomenon that is likely to motivate students may meet the criterion.

Providing students with a sense of purpose for a whole unit or chapter is not always possible (for example, there may not be a single question or problem that is broad enough to foreshadow all learning goals in the unit) or even desirable (for example, providing a purpose on a large scale can lead to a complex sequence of activities that is too demanding on the memory of younger students). In such cases, it may be sufficient for the lesson/activity to frame sections within a unit rather than the whole unit or chapter.

Indicators of meeting Criterion I.1:
1. The purpose is presented to students explicitly (or implicitly through a problem, question, or representation).
2. The purpose is likely to be comprehensible to students.
3. The purpose is likely to be interesting and/or motivating to students.
4. Students are given an opportunity to think about and discuss the purpose.
5. Most activities or lessons are consistent with the stated purpose.

I.2 Conveying Lesson Purpose: Does the lesson/activity convey the purpose of each activity or lesson and its relationship to others?

Clarification:
The question is whether the purpose of individual activities or lessons (as opposed to the whole unit) is made apparent to the students and whether there are logical transitions and connections between activities or lessons. If a classroom visitor asked students what they were doing and why, is there reason to think they would know?

The purpose of individual activities or lessons could be brought out through the text, teacher comments (suggested in the lesson/activity), and/or student responses to questions. For example, the purpose of gathering data about the heights of students in the class (to learn about dispersion of data) might be brought out by text explanation, teacher explanation, or by the students coming up with a description of the spread of data.

Indicators of meeting Criterion I.2:
1. The lesson/activity conveys or prompts teachers to convey the purpose of each activity or lesson to students.
2. Each activity encourages each student to think about the purpose of the activity or lesson.
3. The lesson/activity conveys or prompts teachers to convey to students how each activity or lesson relates to the other activities.
4. The lesson/activity periodically engages students in thinking about what they have learned so far and what they need to learn/do next.

I.3 Justifying Sequence of Activities: Does the lesson/activity involve students in a logical or strategic sequence of activities (versus a collection of activities) that build toward understanding of the ideas in the unit or chapter purpose?

Clarification:
The issue here is whether there is a logical or strategic sequence of activities in the lesson/activity and whether this logic or strategy is made explicit to the teacher or just inferred by the reviewer. A rationale or implicit reason for the sequence of activities should be clear, providing the teacher and students with a sense of making progress toward the purpose of the unit or chapter.

Indicators of meeting Criterion I.3:
1. The lesson/activity provides a rationale for the overall sequence of activities or lessons.
2. If no rationale for the overall sequence of activities or lessons is provided, the reviewer can identify one.
3. The sequence of activities reflects the stated or inferred rationale or purpose.

Category II: Building on Student Ideas about Mathematics

Fostering better understanding in students requires taking time to attend to the ideas they already have, both ideas that are incorrect and ideas that can serve as a foundation for subsequent learning. Such attention requires that teachers be informed about prerequisite ideas/skills needed for understanding a learning goal and what their students' initial ideas

are—in particular, the ideas that may interfere with learning mathematics. Moreover, teachers can help address students' ideas if they know what is likely to work. This category examines whether the lesson/activity contains specific suggestions for identifying and addressing student ideas.

II.1 Specifying Prerequisite Knowledge. Does the lesson/activity specify and address prerequisite knowledge/skills that are necessary to the learning of the learning goal?

Clarification:
This criterion refers to (a) prerequisites to concepts or skills in the learning goal examined, and (b) prerequisites to activities used in the lesson/activity to teach the concepts or skills in the learning goal examined.

(a) Understanding the ideas in learning goals often requires that students first understand some other "prerequisite" concepts or skills. For example, knowing what a prime factor is is prerequisite to learning how to find the common denominator of fractions.

(b) In addition to prerequisites to specific ideas in learning goals, additional prerequisites may arise from the specific activities used to teach them. For example, consider the learning goal: "Spreading data on a number line helps to see what the extremes are, where they pile up, and where the gaps are. A summary of the data includes where the middle is and how much spread is around it." There are no learning goals that are prerequisite to the ideas in this learning goal. However, as the students work through a curriculum lesson/activity, they may be engaged in investigations that target this learning goal and involve measurement. In these cases, the lesson/activity should not take for granted that students will have developed measuring skills. Teachers should be alerted to this prerequisite (measurement skills) and encouraged to support their students in developing measurement skills.

Responding to the "prerequisites" criterion involves (a) making a list of prerequisite concepts and/or skills, (b) examining whether the lesson/activity alerts to any prerequisite ideas on the list and if so which ones, (c) examining whether the lesson/activity has in fact adequately addressed the prerequisites in the same or earlier units, and (d) examining whether the lesson/activity helps students make connections between learning goals and their prerequisites. While a stand-alone unit should not be faulted for not addressing prerequisite ideas or skills, it should be expected to make connections between learning goals and their prerequisites.

Indicators of meeting Criterion II.1:
1. The lesson/activity makes explicit what the specific prerequisite ideas or skills are, if any exist.
2. The lesson/activity addresses the identified prerequisites in the same unit or in earlier units.
3. The lesson/activity makes connections between learning goal ideas and their prerequisites (even if the prerequisites are addressed elsewhere).

II.2 Alerting Teacher to Student Ideas. Does the lesson/activity alert teachers to

commonly held student ideas (both troublesome and helpful) such as those described in *Learning goals for Science Literacy* Chapter 15: The Research Base?

Clarification:
Researchers have identified ideas that students have in several content areas. The issue here is whether the lesson/activity informs teachers about students' commonly held ideas in the topic areas the lesson/activity addresses. This information can help teachers (a) understand better their own students' ideas, (b) decide what ideas to build on and what changes to promote, or (c) if the lesson/activity is already designed in ways that build on or attempt to change students' commonly held ideas, to better understand the rationale and purpose behind designed strategies and activities.

Responding to this question involves examining (1) whether there is research on commonly held student ideas in the topic area/s that the lesson/activity addresses, (2) whether the lesson/activity alerts teachers to such ideas, and (3) whether the lesson/activity accurately represents research findings. Summaries of research on students ideas in mathematics (such as those included in *Learning goals* Chapter 15: The Research Base or the NCTM *Research Ideas for the Classroom* series) will be helpful to reviewers who will want to know what ideas students typically have about the topics that the curriculum lesson/activity they are examining addresses. If there is no research on student ideas in the topic area/s that the lesson/activity addresses, the lesson/activity should not be faulted for not addressing this criterion.

Indicators of meeting Criterion II.2:
1. The lesson/activity lists, conveys, or identifies specific commonly held ideas that are relevant to the learning goal (rather than just to relevant difficult topics).
2. The lesson/activity clarifies/explains commonly held ideas.
3. The lesson/activity explains or refers to commonly held ideas in an accurate way.

II.3 Assisting Teacher in Identifying Ideas. Does the lesson/activity include suggestions for teachers to find out what their students think about familiar situations related to a learning goal before the mathematical ideas are introduced?

Clarification:
Teachers need guidance in identifying students' ideas in these unresearched areas. But even in areas in which there is research on commonly held student ideas, teachers need help in identifying what proportion of their own students hold these ideas as well as other more idiosyncratic ideas.

Responding to this criterion involves examining not only whether the lesson/activity encourages teachers to find out students' ideas but also whether it provides specific suggestions for how to do so. Suggestions may include providing (a) tasks in which students make predictions and give their own descriptions and explanations of concepts or skills; (b) tasks in which students are asked to represent their understandings in drawings; (c) tasks in which students are asked to interpret information (for example the solution to an algorithm related to ideas in the learning goal), discuss connections with related topics, or discuss alternative solutions, or justifications; or (d) tasks which ask

students about the meaning of specific terms and/or probe for understanding of important relationships between concepts.

Responding to the criterion also involves examining the quality of the suggestions provided. Tasks should not focus exclusively on identifying students' meaning for terms. While including such tasks is useful, it is important to look for tasks in which students make predictions and/or give explanations of concepts or procedures. It is important that tasks make sense to students who have never studied the topic and include questions posed in ways meaningful to the students who are not familiar with the mathematical vocabulary. It is also important that the lesson/activity encourages teachers to use probing questions to clarify what students mean or to get more information about students' thinking.

Indicators of meeting Criterion II.3:
1. The lesson/activity includes specific questions or tasks to assist the teacher in identifying the ideas students have before they study the learning goal.
2. The questions or tasks are likely to be comprehensible by students before they become familiar with the concepts, procedures, or vocabulary.
3. The lesson/activity includes questions or tasks that ask students to make predictions or give explanations of concepts or procedures (vs. focus primarily on identifying students' meaning for terms).
4. The lesson/activity suggests how teachers can use questions or tasks to understand students' thinking and level of understanding.

II.4 Addressing Misconceptions. Does the lesson/activity explicitly address commonly held student ideas?

Clarification:
The issue here is whether the lesson/activity includes questions or activities that address students' commonly held ideas (both concepts or skills that are incorrect and those that can serve as a foundation for subsequent learning). For example, the lesson/activity may include experiences that help students change their ideas by providing activities that challenge students' predictions or explanations, or prompt students to react to commonly held misconceptions and resolve differences between these misconceptions and the correct ideas. Alternatively, the lesson/activity may include experiences that extend common student concepts or skills that have limited scope. Pointing out misconceptions and telling students that they should avoid them does not adequately address this criterion. Serious difficulties, either with concepts or with skills, are not generally successfully addressed by telling students they are wrong and providing them with the "right answer."

In addition to providing specific suggestions to teachers about how to address commonly held student ideas reported in the research literature, lesson/activitys can be helpful by including suggestions to teachers about how to take into account their own students' ideas. Addressing this aspect of the criterion may involve suggesting general strategies that teachers can use to build on or change students' ideas, and providing examples of

how these strategies can be implemented in the classroom. For example, teachers can be encouraged to probe students' ideas further, juxtapose them with other students' ideas, encourage students to compare their ideas on a topic before and after instruction on the topic, etc.

Indicators of meeting Criterion II.4:
1. The lesson/activity explicitly addresses commonly held ideas related to the learning goal (if there is research on these ideas).
2. The lesson/activity includes questions, tasks, or activities that are likely to help students progress from their initial ideas, for example, by:

> (a) explicitly challenging students' ideas, for example, by comparing their predictions to what actually happens.
>
> (b) prompting students to contrast commonly held ideas and the correct concept or procedure, and resolve differences between them.
>
> (c) extending correct commonly held ideas that have limited scope.

3. The lesson/activity suggests general strategies for addressing student ideas related to the learning goal.

Category III: Engaging Students in Mathematics

Much of the point of mathematics is finding patterns and modeling ideas and relationships in terms of a small number of generalizations or ideas. For students to appreciate the power of mathematics, they need to have a sense of the range and complexity of ideas and applications that mathematics can explain or model. "Students need to get acquainted with the things around them—including devices, organisms, lesson/activitys, shapes, and numbers—and to observe them, collect them, handle them, describe them, become puzzled by them, ask questions about them, argue about them, and then try to find answers to their questions." (*Science for All Americans*, p. 201)

III.1 Providing Variety of Contexts. Does the lesson/activity provide experiences with mathematics in multiple, different contexts?

Clarification:
Mathematicians and others construct and use mathematical knowledge to describe, explain, predict, and design real-world objects, systems, or events as well as abstract relationships. Therefore, mathematical ideas need to be connected to meaningful problems, situations, and the real world. The question is whether the lesson/activity provides a sufficient number of problems, experiences, or events in a variety of contexts to support the ideas put forth in learning goals. The lesson/activity can provide experiences with problems, situations, systems, or events directly through hands-on activities or demonstrations (firsthand experiences) or indirectly, through the use of text, graphs, diagrams, computer screens, videos, pictures, models, etc.

Indicators of meeting Criterion III.1:
1. The experiences with objects, applications, and lesson/activitys are "right on target" in addressing the content of the learning goal.
2. The lesson/activity provides an appropriate variety of experiences with objects, applications, and lesson/activitys.

III.2 Providing Firsthand Experiences. Does the lesson/activity include activities that promote firsthand experiences with the learning goal ideas, when practical?

Clarification:
Students can learn more readily about things that are tangible and accessible to their senses; thus students, especially younger ones, will benefit most from firsthand experiences with the objects, problems, or events to which the mathematical knowledge in a learning goal refers. Providing students with some firsthand experiences (e.g., hands-on activities, problem solving, or measuring) is important, provided such experiences are practical. When such experiences are not practical (for example, providing firsthand experiences with measuring the height of a mountain), students can encounter objects and events indirectly, through the use of videos, pictures, models, etc.

However, it is neither necessary nor optimal that all experiences provided are firsthand. (For example, once students have had some firsthand experience with flipping coins to find probabilities, providing them with examples of other events that have finite outcomes would likely be adequate.) If all experiences provided to students were firsthand, it would limit the number of examples that could be provided (see previous criterion, Providing Variety of Contexts). Moreover, students should not be asked to reason only about ideas they see firsthand, when in real life they will also encounter problems indirectly.

Indicators of meeting Criterion III.2:
1. The activities, whether firsthand or not, provide experiences (e.g., text, pictures, video) that give students meaningful connections of the concept or skill to their knowledge.
2. An appropriate number of experiences with ideas are firsthand experiences.
(The number of firsthand experiences that are appropriate depends on the age level of the students and the difficulty of the learning goal.)
3. The firsthand experiences are efficient when compared to other firsthand experiences that could be used. (Efficiency of an experience is judged by the time and cost of the experience in relation to its value.)

Category IV: Developing Mathematical Ideas

Science for All Americans includes in its definition of mathematics literacy a number of important yet quite abstract ideas—e.g., symbolic representation, patterns and relationships, summarizing data. Such ideas cannot be readily discovered in the real world; the ideas themselves were developed over many hundreds of years as a result of considerable discussion and debate about the existence and logic of laws of mathematics

and proofs of theorems. Mathematics literacy requires that students see the link between concepts and skills, see mathematics itself as logical and useful, and become skillful at using mathematics. This category includes criteria to determine whether the lesson/activity expresses and develops ideas in ways that are accessible and intelligible to students, and to demonstrate the usefulness of the concepts and skills in varied contexts.

IV.1 Justifying Importance of Learning goal Ideas. Does the lesson/activity suggest ways to help students develop a sense of the importance and validity of mathematical concepts or procedures?

Clarification:
This criterion highlights the importance of including some instances in the curriculum where an argument is developed in support of the concepts, skills, or strategies presented. There are both logical and psychological reasons for expecting a lesson/activity to provide students with a sense of why ideas make sense and of why mathematicians are interested in them.

First, an understanding of the link between hypotheses and argument is itself a literacy goal. *Science for All Americans* includes in its definition of science literacy a basic knowledge of the nature of mathematics—both its logic and creativity—and its central role in human endeavor. Learning about the nature of mathematics can be studied in the context of learning about nearly any discipline, including students' own discoveries.

Given the time it takes to properly develop an argument for ideas and the increased level of sophistication required for understanding both the evidence and the arguments, there is a limited number of ideas for which an evidence-based argument is required for literacy. However, it is possible that some concepts or procedures themselves are sufficiently difficult for students to understand—e.g., long division, infinite decimals—that a case needs to be made for students to find them plausible. The case might involve examining whether the concept fits well with other concepts, explains several relationships, and predicts new observations, and how it compares to other explanations of the same observations. If such a case is likely to be too difficult for most students to understand, then students should at least be informed that they are being asked to take an idea on faith. When a lesson/activity does not attempt to make a case, reviewers should comment on (a) whether or not a case ought to have been made and, if so, why and (b) whether or not the lesson/activity makes explicit that a case is not being built.

Indicators of meeting Criterion IV.1:
1. The lesson/activity builds a case for the mathematical importance of the learning goal.
2. The lesson/activity builds a case for the validity of the mathematical ideas.
3. The lesson/activity builds a case for the learning goal that is likely to be comprehensible to students.
4. The lesson/activity engages students in considering a case for the validity and importance of learning goal concepts or skills.

IV.2 Introducing Terms and Procedures. Does the lesson/activity introduce terms and procedures only in conjunction with experience with them and only as needed to

facilitate thinking and promote effective communication?

Clarification:
Understanding, rather than simply memorizing vocabulary or algorithms, should be the main purpose of mathematics teaching. In mathematics, many terms refer to concepts. For students to understand a concept, they should be able to describe its properties, give examples and non-examples of it, and eventually give a definition. Algorithms are important in themselves, as well as providing efficient ways to solve problems. Students should have opportunities to apply the concepts or procedures in problems and reasoning.

Indicators of meeting Criterion IV.2:
1. The lesson/activity limits the use of terms and procedures.
2. The lesson/activity introduces mathematical vocabulary or procedures in conjunction with experiences, rather than having students simply memorize definitions or procedures.
3. The lesson/activity provides appropriate examples or meaningful applications of the terms or procedures.

IV.3 Representing Ideas Accurately. Does the lesson/activity include accurate and comprehensible representations of mathematical concepts, procedures, and relationships?

Clarification:
This question highlights the importance of using accurate representations to make (abstract) ideas intelligible to all students. Different representations highlight different aspects of an idea and provide a variety of opportunities for the idea to connect to other students' ideas and become embedded in a student's knowledge system. Possible modes of representation include drawings, diagrams, graphs, images, analogies and metaphors, models and simulations, and role-playing. Representations need to be clear so that students can understand fairly quickly what ideas are being represented and how. In addition, because representations typically highlight only some aspects of an idea, care must be taken that they represent the real thing as accurately as possible (or they involve students in considering which aspects of the real thing are represented by the model and which are not).

Indicators of meeting Criterion IV.3:
1. The lesson/activity includes accurate representations.
2. The lesson/activity includes comprehensible representations (depending on the students' grade level and the difficulty of the learning goal).
3. The lesson/activity includes an appropriate number and variety of accurate and comprehensible representations.

IV.4 Connecting Learning goal Ideas. Does the lesson/activity explicitly draw attention to appropriate connections among learning goal ideas?

Clarification:
This criterion emphasizes connections among learning goal ideas, concepts, and skills

(rather than connections between activities, which are examined in Category I). Some kinds of connections can be classified as belonging to one of the following general types:

> (a) One concept or skill may be an instance of a generalization (e.g., the sum of the angles of a rectangle illustrates the general sum of the angles of a polygon).
>
> (b) One concept or skill might be analogous to another idea (e.g., adding rational fractions is like adding rational expressions in that they both require a common denominator).
>
> (c) A concept or skill may show up in several fields or contexts (e.g., the number pi can be the ratio of the circumference to diameter, the radian measure of a semicircle, or the sum of a series).

Other connections are more unique to particular ideas (e.g., linking slope of a line to the tangent of a curve).

Responding to this criterion involves looking to see whether any general or unique connections are essential, which requires identifying what such connections might be. Growth of Understanding maps provide a rich source of potential connections. A set of maps on about 50 topics will be available in *Atlas of Science Literacy*. When a map is not available for a topic, the "Also-See" box in *Learning goals for Science Literacy* may be helpful in identifying conceptual connections among ideas. Reading *Science for All Americans* may also be helpful in identifying conceptual connections, since several important connections among ideas are made in the prose.

Indicators of meeting Criterion IV.4:
1. The lesson/activity notes connections among specific learning goal ideas (rather than just among general topics).
2. The lesson/activity adequately explains or develops the identified connections.
3. The lesson/activity engages students in making and/or explaining the identified connections.

IV.5 Demonstrating/Modeling Procedures. Does the lesson/activity demonstrate/model (or include suggestions for teachers on how to demonstrate/model) skills or the use of knowledge?

Clarification:
Among the ways literate adults use their knowledge and skills are to describe and explain phenomena, to solve practical problems, and to consider alternative positions on issues. Hence students should learn to use their knowledge and skills in these ways. In order for students to know the type and level of performance expected for a skill or an application of conceptual knowledge, they need to see examples. This is particularly important for complex behaviors such as explaining how to solve problems, developing a generalization, argument, or proof, or carrying out a complex procedure or algorithm. Demonstrating or Modeling a skill involves (a) an expert's demonstrating or modeling the skill, (b) providing running commentary about important aspects to note about the

performance or demonstration, and (c) providing criteria for judging a good performance. Demonstrating or Modeling how knowledge might be used, for example, to solve problems or construct a proof, is similar.

Responding to this criterion involves examining whether (a) demonstrating/modeling is carried out by the text or other accompanying lesson/activitys (e.g., software, video), or (b) the lesson/activity includes suggestions to teachers about how to go about demonstrating/modeling skills or use of knowledge in their classrooms.

Indicators of meeting Criterion IV.5:
1. The lesson/activity demonstrates (or instructs teachers how to demonstrate) the expected procedure or performance.
2. The demonstration is clear and comprehensible.
3. The lesson/activity provides commentary that points to particular aspects of the demonstration and/or provides justifications or explanations for the steps taken.

IV.6 Providing Practice. Does the lesson/activity provide tasks or questions for students to practice skills or use knowledge in a variety of situations?

Clarification:
An important part of learning mathematics consists of giving students many opportunities to use their skill or knowledge, in particular giving them opportunities to practice using mathematical knowledge and skills in describing objects, relationships, and events, solving problems, and applying knowledge in new situations or contexts. Moreover, literacy means that people will be able to draw upon and use their understanding of mathematics when they encounter events that do not come with labels such as "algebra," "geometry, " or "statistics" but in political arguments, discussions of literature, and walks on the beach. Therefore, students will need practice in making connections to new situations. Providing students with opportunities to practice only calculating answers to predictable exercises does not adequately address this criterion.

Indicators of meeting Criterion IV.6:
1. The lesson/activity includes appropriate practice exercises and tasks on the learning goal.
2. The lesson/activity provides an appropriate number of practice exercises and tasks.
3. The lesson/activity includes a variety of contexts, including everyday tasks and contexts and novel as well as familiar practice tasks.

Category V: Promoting Student Thinking about Mathematics

No matter how clearly lesson/activitys may present ideas, students (like all people) will devise their own meaning, which may or may not correspond to targeted learning goals. Students need to make their ideas and reasoning explicit, hold them up to scrutiny, and recast them as needed. This category includes criteria for whether the lesson/activity

suggests how to help students express, think about, and reshape their ideas to make better sense of the world.

V.1 Encouraging Students to Explain Their Reasoning. Does the lesson/activity routinely include suggestions for having each student express, clarify, justify, and represent his/her ideas and how to get feedback from peers and the teacher?

Clarification:
It is important to provide opportunities for students' thinking to become overt to themselves, the teacher, and other students. By stating (clarifying, justifying, and representing) their ideas, in writing, drawing, or speaking, students become more aware of what they think. This may stimulate making explicit connections between their ideas and the ideas presented by the text or the teacher, and/or questioning of their ideas (if relevant). Exchange of ideas in small groups or a large group discussion may make students aware of the range of ideas that exist and may provoke students to reconsider their own ideas in light of others. Feedback from the teacher or other peers is necessary to help students understand their mistakes and improve the quality of their descriptions, explanations, or designs.

Responding to the first part of the criterion involves examining whether the lesson/activity prompts (or encourages teachers to prompt) students to express their ideas either orally or in writing. It also involves examining whether the lesson/activity has opportunities for each student to express his or her ideas. Responding to the second part of the criterion includes examining whether the lesson/activity includes specific suggestions to help the teacher provide explicit feedback, includes text that directly provides students with feedback on the adequacy of their ideas, or provides teachers with strategies they can use to ensure that each student in the class receives feedback.

Indicators of meeting Criterion.V.1:
1. The lesson/activity encourages students to express their ideas about the learning goal.
2. The lesson/activity encourages students not only to express but also to clarify, justify, interpret, or represent learning goal ideas.
3. The lesson/activity provides (or includes suggestions to help the teacher to provide) explicit feedback to students about their ideas.
4. The lesson/activity includes suggestions to the students or teacher on how to use student responses to diagnose errors or difficulties, address errors or difficulties, or further develop students' ideas about the learning goal.

V.2 Guiding Interpretation and Reasoning. Does the lesson/activity include tasks and/or question sequences that guide student interpretation and reasoning about learning goal concepts, skills, and relationships?

Clarification:
Experiences with hands-on lesson/activitys, problems, and examples of mathematical ideas are useful but not sufficient. Students need time, opportunities, and guidance to make sense of these experiences. If the students are turned loose to do exercises or problems on their own, very little happens except for a small number of students. The

activities need to be guided with sequences of questions that lead students to make relevant generalizations and understand relationships. Similarly, students need time, opportunities, and guidance to make sense of things they read and ideas they are introduced to.

Responding to this criterion involves examining whether the lesson/activity includes (in the teacher's guide or student books) specific, carefully chosen and sequenced tasks or questions that are likely to support students' thinking about exercises, problems, and investigations. Good tasks and questions frame important issues, help students relate their previous experiences to the mathematical ideas, anticipate common student difficulties or misconceptions, and focus on important generalizations and procedures.

Indicators of meeting Criterion V.2:
1. The lesson/activity includes specific and relevant tasks and/or questions for activities related to the learning goal.
2. The lesson/activity includes connected sequences (rather than only collections) of questions or tasks.
3. The questions or tasks guide student interpretation and reasoning through approaches such as:

(a) framing, introducing, or developing important ideas,
(b) helping students to relate their own experiences to mathematical ideas, or
(c) anticipating or eliciting common difficulties or student misconceptions.

V.3 Encouraging Students to Think about What They've Learned. Does the lesson/activity suggest ways to have students check their own progress?

Clarification:
This criterion highlights the importance of having students look back at the progress of their thinking and learning. Monitoring one's understanding and realizing which ideas one does not understand can shift some of the responsibility for learning to the students and may elicit their attempts to understand as a result.

Responding to this criterion involves examining whether the lesson/activity includes questions or tasks that prompt students to monitor their understanding, or includes suggestions to teachers on how to encourage students to do so. For example, What was confusing to you today? How does the new knowledge compare with what you used to think? What do you think you understand and where do you need to work more? Encouraging students to monitor their understanding should also include (when appropriate) questions on how and why students changed their ideas. For example, Did you change any of your ideas today? What evidence convinced you to do so?

Indicators of meeting Criterion V.3:
1. The lesson/activity engages (or provides specific suggestions for teachers to engage) students in monitoring their progress toward the learning goal.
2. The lesson/activity asks students to think about how their ideas have developed or changed.

3. The lesson/activity gives students an opportunity to revise their initial ideas about the learning goal based on what they have learned.

Category VI: Assessing Student Progress in Mathematics

Assessment provides information to students about what is important and to teachers about what has been learned. Just as important, assessment provides information to both the students and the teacher about adjustments that should be made in learning and instruction. Because assessment is so important to the teaching-learning process, it must match the mathematics learning goal of the curriculum lesson/activitys. Further, assessments must address the range of skills, applications, and contexts that reflect what students are expected to learn. All of this is possible only if assessment takes place throughout instruction, not only at the end of a chapter or unit.

VI.1 Aligning Assessment. Are assessment items or tasks included that match the ideas, concepts, or skills of the learning goal?

Clarification:
This criterion highlights the necessity of including assessment items for each learning goal that is important in the lesson/activity. To judge whether the items provided match the standard, the same procedure should be followed as in examining the content match between an activity and a learning goal. That is, examine whether the assessment item addresses the substance rather than only the topic of the learning goal, the level of sophistication of the learning goal, and what part of the learning goal is assessed.

Indicators of meeting Criterion VI.1:

1. The lesson/activity provides at least one assessment task that addresses the specific ideas of the learning goal (assessment item should not be answerable by reading comprehension, general intelligence, or test-wiseness alone).
2. The assessment items that do address the learning goal require no other, more sophisticated, ideas.
3. If the lesson/activity provides a test that is given to the students, an appropriate number of assessment items are content-matched to the learning goal. If the lesson/activity provides a bank of assessment items that teachers select from, an appropriate proportion of assessment items are content-matched to the learning goal.

VI.2 Formal Assessment/Assessment through Applications. Does the lesson/activity include assessment tasks that require application of learning goal ideas, concepts, or skills and avoid allowing students a trivial way out, like using a formula or repeating a memorized term or rule without understanding?

Clarification:
Rather than checking whether students have memorized certain bits of information, assessment needs to test students' mathematical understanding, reasoning, and the application of knowledge. In addition, it needs to include tasks that engage students in

activities similar to those they will engage in in their lives outside the classroom. Literate persons use mathematical knowledge to describe, explain, and predict real world phenomena, solve a practical problem, or discuss issues. Accordingly, assessment tasks need to engage students in descriptions, explanations, predictions, design, and discussion of issues. This, however, does not necessarily dictate the format that the assessment should include. For example, assessments of students' use of knowledge to explain a concept could include a "multiple choice" or "constructed response" format.

Indicators of meeting Criterion VI.2:
1. The lesson/activity provides assessment tasks that focus on application of learning goal ideas.
2. The lesson/activity includes assessment tasks that are familiar as well as tasks that are novel or non-routine.
3. If the lesson/activity provides a test that is given to the students, an appropriate number of assessment items focus on application. If the lesson/activity provides a bank of assessment items that teachers select from, an appropriate proportion of items or tasks focus on application.

VI.3 Using Informal/Embedded Assessment. Are some assessments embedded in the curriculum along the way, with advice to teachers as to how they might use the results to choose or modify activities?

Clarification:
This criterion highlights the need for assessment to be in the service of instruction to guide teaching and learning. The criterion requires that lesson/activitys include assessments that can be used as diagnostic or formative instruments, which help determine learners' needs, rather than largely as instruments for grading students at the end of a unit or chapter.

Responding to this question involves examining whether the lesson/activity (a) provides assessment tasks only at the end of a unit of study to help grade student achievement, or also along the way to help monitor student progress, (b) encourages and provides guidance to teachers about how to probe beyond students' first response to clarify and further understand student answers, and (c) encourages teachers to use the information from the assessments to make instructional decisions about what ideas need to be addressed by further activities with the whole group or smaller groups of students.

Indicators of meeting Criterion VI.3:
1. The lesson/activity uses embedded assessment as a part of the instructional strategy or design.
2. The lesson/activity includes assessments that provide opportunities, encouragement, or guidance for students on how to further understand learning goal ideas.
3. The lesson/activity includes suggestions or guidance for teachers on how to probe students' understanding of learning goal ideas.
4. The lesson/activity provides specific suggestions to teachers about how to use the

information from the embedded assessments to make instructional decisions about what ideas need to be addressed by further activities.

Category VII. Enhancing the Mathematics Learning Environment

Several other important considerations are involved in the selection of curriculum lesson/activitys—for example, the help they provide teachers in encouraging student curiosity and creating a classroom community where all can succeed, or attractiveness. These can influence student learning or even whether the lesson/activitys are used appropriately by the teacher and students. The criteria listed in this category provide reviewers with the opportunity to comment on these and other important features.

VII.1 Providing Teacher Content Support. Does the lesson/activity help teachers improve their understanding of mathematics and its applications?

Clarification:
The issue here is whether the lesson/activity includes a "content background" section or other features that help teachers develop their understanding of the mathematical knowledge addressed in the lesson/activity. Responding to this question involves commenting on the quality of the support, not merely on whether such support is included. Just providing teachers with a list of resources that may enhance their understanding of mathematics does not adequately address this question. A minimum requirement is that such lists are annotated to describe the resources and specify what can be learned from them.

Indicators of meeting Criterion VII.1:
1. The lesson/activity provides content information or recommends resources for improving specific skills or understanding of particular ideas.
2. The lesson/activity provides content information that is in the form that is useful and appropriate for teachers, no matter what their background knowledge.
3. The lesson/activity indicates how the ideas or skills are relevant and important to teaching the lesson/activity to students.

VII.2 Establishing a Challenging Classroom. Does the lesson/activity help teachers to create a classroom environment that welcomes student curiosity, rewards creativity, encourages a spirit of healthy questioning, and avoids rigidity?

Clarification:
Responding to this criterion involves examining whether teachers are given guidance to (a) encourage students to raise questions about the lesson/activity being studied and suggest productive ways for finding answers, (b) use activities in which students' creativity and imagination will pay off, (c) respect and value students' ideas, and (d) avoid conveying the impression that they themselves or the textbooks are absolute authorities whose conclusions are always correct. In addition, the criterion involves examining whether the lesson/activitys give a vision of what the curriculum might look

like in action (i.e., teacher hints and suggestions, dialogue boxes, vignettes, or video clips that show desirable student teacher interactions).

Indicators of meeting Criterion VII.2:
1. The lesson/activity provides opportunities for students to express their curiosity or creativity.
2. The lesson/activity provides occasions for students to take risks and ask questions.
3. The lesson/activity suggests how to encourage students to weigh and challenge their own and others' ideas.
4. The lesson/activity avoids sending a message that mathematics consists only of rules and single correct answers.

VII.3 Supporting All Students. Does the lesson/activity help teachers to create a classroom that encourages high expectations for all students, enables all students to experience success, and provides all students a feeling of belonging in the mathematics classroom?

Clarification:
Several pedagogical criteria presented in previous categories highlight the need for lesson/activitys to incorporate principles of teaching and learning that are likely to promote mathematics understanding for all students. This question highlights the importance of reviewing curriculum lesson/activitys for features that might distract or impede the progress of females, minorities, students whose first language is not English, students with disabilities, or others from the intended work. Further, the criterion requires that lesson/activitys provide specific suggestions and resources for encouraging all students to be able to learn mathematics and express their competence and performances during instruction and in assessment tasks.

Indicators of meeting Criterion VII.3:
1. The lesson/activity avoids stereotypes or language that might be offensive to a particular group.
2. The lesson/activity illustrates the contribution or participation of women, minorities, and persons with disabilities to mathematics-related fields.
3. The lesson/activity suggests alternative formats for students to develop or express their mathematics knowledge during instruction and assessment.
4. The lesson/activity includes specific suggestions on how teachers can modify activities for students with special needs, interests, or abilities.

Appendix C

Growth across the Grades: Aiming for Focus and Coherence

Each of these ten Standards applies across all grades, prekindergarten through grade 12. The set of Standards, which are discussed in detail in chapters 4 through 7, proposes the mathematics that all students should have the opportunity to learn. Each Standard comprises a small number of goals that apply across all grades—a commonality that promotes a focus on the growth in students' knowledge and sophistication as they progress through the curriculum. For each of the Content Standards, chapters 4 through 7 offer an additional set of expectations specific to each grade band.

The Table of Standards and expectations in the appendix highlights the growth of expectations across the grades. It is not expected that every topic will be addressed each year. Rather, students will reach a certain depth of understanding of the concepts and acquire certain levels of fluency with the procedures by prescribed points in the curriculum, so further instruction can assume and build on this understanding and fluency.

Even though each of these ten Standards applies to all grades, emphases will vary both within and between the grade bands. For instance, the emphasis on number is greatest in prekindergarten through grade 2, and by grades 9–12, number receives less instructional attention. And the total time for mathematical instruction will be divided differently according to particular needs in each grade band—for example, in the middle grades, the majority of instructional time would address algebra and geometry. Figure 3.1 shows roughly how the Content Standards might receive different emphases across the grade bands.

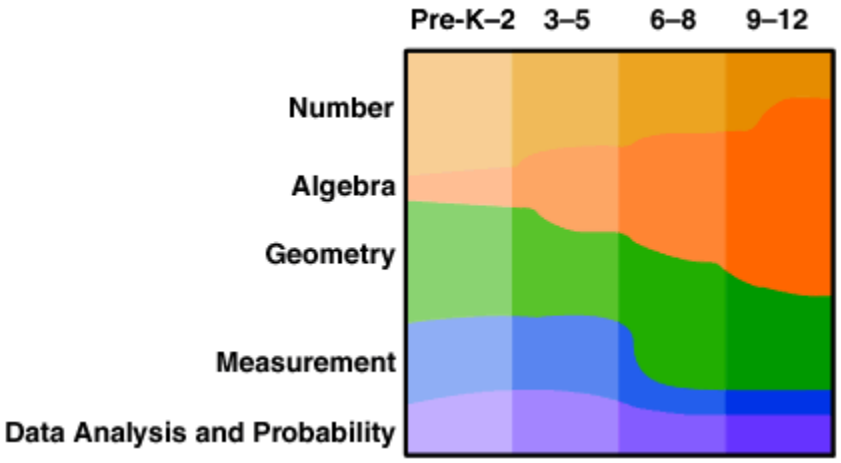

Fig. **3.1.** The Content Standards should receive different emphases across the grade bands.

This set of ten Standards does not neatly separate the school mathematics curriculum into nonintersecting subsets. Because mathematics as a discipline is highly interconnected, the areas described by the Standards overlap and are integrated. Processes can be learned within the Content Standards, and content can be learned within the Process Standards. Rich connections and intersections abound. Number, for example, pervades all areas of mathematics. Some topics in data analysis could be characterized as part of measurement. Patterns and functions appear throughout geometry. The processes of reasoning, proving, problem solving, and representing are used in all content areas.

The arrangement of the curriculum into these Standards is proposed as one coherent organization of significant mathematical content and processes. Those who design curriculum frameworks, assessments, instructional materials, and classroom instruction based on *Principles and Standards* will need to make their own decisions about emphasis and order; other labels and arrangements are certainly possible.

Appendix D
Balance of Representation
More information can be found at
http://www.state.me.us/education/lsalt/LAS/resources.htm

PURPOSE for establishing Balance of Representation

In order for an assessment system to have content validity, it must be aligned with articulated standards. In the case of Maine's *Learning Results*, that alignment occurs at the Performance Indicator level. One aspect of alignment, Balance of Representation, dictates that the learning expectations (performance indicators) must be assessed in proportion to their **scope** and **importance**. Simply stated, big important ideas should receive greater emphasis than smaller, more peripheral ideas.

A systematic procedure for establishing Balance of Representation maps the "topography" of each discipline, identifying the relative mountains and valleys within the landscape of the *Learning Results*. The procedure is guided by knowledge of the discipline, knowledge of curriculum and instruction, and knowledge of students and developmental considerations.

Mathematics Content Standards Weightings